From the private to the public: The road from Zurich (1897) to Madrid (2006)

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Abstract. We review the history of the International Congresses of Mathematicians, from Zurich 1897 to Madrid 2006, mentioning some of the most significant personages and events (scientific as well as political) of such meetings (as a matter of fact, that history did not begin 1897 in Zurich, but in 1893 at the Chicago World's Columbian Exposition). We report on Felix Klein, Henri Poincaré, David Hilbert, Vito Volterra, Emmy Noether, Ludwig Prandtl, Laurent Schwartz, Klein's *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, and the role played by the Mathematical Tripos in the education at Cambridge University during the 19th and the first decades of the 20th century (and its influence in the design of the Cambridge 1912 Congress). Some of the further topics discussed here are how the two world wars affected the congresses, the creation of the Fields medals, as well as some of the main changes that mathematics has experienced, internally (i.e., in what it refers to problems, ideas and theories) as well as institutionally (including its manifold connexions with society) during the period covered by the existence of the International Congresses of Mathematicians.

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"What was formerly begun by a single mastermind, we now must seek to accomplish by united efforts and cooperation". The author of these prophetic words was Felix Klein, the distinguished German mathematician, and they were stated at the "World Congress" of mathematicians that took place, from the 21st to the 26th of August, as part of the 1893 Chicago World's Columbian Exposition (Klein was one of the commissioners of the German university exhibit at the Exposition). Forty-five mathematicians attended that meeting, of which only four were foreigners: the Germans Klein and Eduard Study (the only ones who delivered lectures), the Austrian Norbert Hertz, and the Italian Bernard Paladini, although Charles Hermite, David Hilbert, Adolf Hurwitz, Hermann Minkowski, Max Noether, Salvatore Pincherle and Heinrich Weber supported the congress by submitting papers *in absentia*.[1]

The leading role of Klein in promoting in the New World the spirit of international mathematical collaboration and interchanges (also, of course, German culture) is

[1] The proceedings were published as *Mathematical Papers Read at the International Mathematical Congress Held in Connection with the World's Columbian Exposition Chicago 1893*, E. H. Moore *et al*., eds. (Macmillan & Co., New York 1896). About the congress, and Klein's influence in America, see Karen Hunger Parshall and David E. Rowe, *The Emergence of the American Mathematical Research Community, 1876–1900: J. J. Sylvester, Felix Klein, and E. H. Moore* (American Mathematical Society/London Mathematical Society, Providence 1994).

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clear. Indeed, after the adjournment of the Chicago congress, the Göttingen professor consented to hold a colloquium on mathematics with those members of the meeting which might wish to participate. The Northwestern University at Evanston, Illinois, tendered the use of rooms for such purpose, and there Klein lectured to 23 American mathematicians (Study was also present) from August 28 until September 9. Alfred Clebsch, Sophus Lie, the shape of algebraic curves and surfaces, the transcendency of the numbers e and π, the solution of higher algebraic equations, hyperelliptic and Abelian functions, and the study of mathematics in Göttingen were among the persons and topics he treated.[2]

Obviously, we cannot consider the Chicago meeting as the first international congress of mathematicians, but it certainly was the forerunner of those meetings, which began four years later in Zurich 1897, attended by less than 200 mathematicians (only 4 of them, by the way, women).

More than a century after Klein's above cited statement, looking back from Madrid 2006 at what has happened in mathematics during all those years, we realise how wise the author of the "Erlangen Program" was. It is true that mathematics can still, perhaps more than any other scientific discipline, be practised with outstanding success in the solitude of a room, by an individual, by a "single mastermind", but it is not less true that its scope has grown in such a way that it needs for its development and health, and very much so, "united efforts and cooperation". That this is so is due not only, nor perhaps mainly, to the increasing difficulty of mathematical problems, but to the incredible widening of the range of fields in which mathematical expertise is needed: from aerodynamics and hydrodynamics to economics and meteorology, from architecture to ecology, not forgetting more traditional sites such as physics, or others that appeared during the XXth century, as, for instance, the design of computers. It is because of this, as well as because of the almost dramatic growth of the attendance to the International Congresses of Mathematicians, that in the title of my exposition appears the expression "From the private to the public."

Before turning to the Zurich Congress, let me point out that mathematics was not the first scientific discipline to organise international meetings. Thus, chemists assembled in the famous 1860 Karlsruhe International Congress of Chemists (again, it was the idea of a German, Friedrich August Kekulé).[3] However, that meeting was not followed by others; that is, there was not the continuity that would characterise the International Congresses of Mathematicians. In this sense, the mathematicians meetings opened a new era of international communication that was adopted by other scientific (and non-scientific) disciplines during the XXth century. (Physicists had to wait until 1911, with the Solvay Conseils, but even then their scope was very different, with only a handful of scientists being invited).

[2] Klein's lectures were published as: Felix Klein, *Lectures on Mathematics* (Macmillan, New York 1894; reprinted by AMS Chelsea, Providence 2000).

[3] *Compte rendu des séances du Congrès international des chimistes réuni à Carlsruhe le 3, 4 et 5 septembre 1860*, "Anlage 8" in vol. I of *August Kekulé*, 2 vols. (Verlag Chemie, Berlin 1929), pp. 671–688; reproduced in Mary Jo Nye, ed., *The Question of the Atom. From the Karlsruhe Congress to the First Solvay Conference, 1860–1911* (Tomash Publishers, Los Angeles 1984), pp. 633–650.

The first International Congress of Mathematicians: Zurich (1897)

The ideals of "united efforts and cooperation" which Klein stated in Chicago were taken up by Hermann Minkowski, then professor at the Zürich Polytechnikum, where he was teaching, among other students, Albert Einstein. In a letter dated November 17, 1896, he wrote to his friend David Hilbert (whom he joined a few years later in Göttingen):[4] "It just occurred to me – for several days we have been meeting in regard to the international mathematical congress." Indeed, an announcement of the proposed meeting was circulated at the beginning of the new year under the names of 22 mathematicians, eleven of which were from Zurich (one of them was C. F. Geiser, another of Einstein's teachers at the Polytechnikum). "After considerable correspondence," the announcement read, "the question of the place of holding the Congress has been decided in favour of Switzerland, as a country peculiarly adapted by situation, relation, and traditions of promoting international interests."

The opening lecture of the congress was given by Henri Poincaré ("Sur les rapports de l'analyse pure et de la physique mathématique"),[5] who, however, could not attend the meeting because of illness (a professor of the Polytechnikum, Jérôme Franel, read his contribution), while the closing one was delivered by Felix Klein ("Zur Frage des höheren mathematischen Unterrichts"). It was quite appropriate that Poincaré and Klein, who had spent years competing in mathematics research, were united in that historical First International Congress of Mathematicians. Only two other plenary lectures were delivered, by Adolf Hurwitz (Zurich) and Giuseppe Peano (Turin).

Paris (1900): Hilbert's famous lecture

Some thought improper Poincaré's lecture in Zurich, although posterity has given it a different status. David Hilbert considered whether he should reply to it when he thought about which topic to talk at the next congress, but Minkowski recommended him not to do so. Instead, he advised him in a letter of January 5, 1900, that it would be "most alluring… to attempt to look into the future, in other words, a characterisation of the problems to which the mathematicians should turn into the future. With this you might conceivably have people talking about your speech even decades from now. Of course, prophecy is indeed a difficult thing."[6] Hilbert followed his suggestion.

The Second International Congress of Mathematicians, held in Paris from August 6 to 12, 1900, with Charles Hermite as *Président d'honneur* and Poincaré as effective

[4] Quoted in Donald J. Albers, G. L. Alexanderson and Constance Reid, *International Mathematical Congresses. An Illustrated History, 1893–1986* (Springer-Verlag, New York 1987, revised edition), p. 4.

[5] Henri Poincaré, "Sur les rapports de l'analyse pure et de la physique mathématique", *Verhandlungen des ersten Internationalen Mathematiker-Kongresses, Zürich, 1897*, F. Rubio, ed. (Teubner, Leipzig 1898), pp. 81–90. Poincaré's lecture has been analysed by Jeremy J. Gray, *The Hilbert Challenge* (Oxford University Press, Oxford 2000), pp. 80–83. Besides of appearing in the proceedings of the congress, Poincaré's lecture was inserted in his book *La valeur de la science* (1905), as well as in *Acta Mathematica* 21, 331–341 (1897)).

[6] Quoted in J. Gray, *The Hilbert Challenge*, *op. cit*., p. 57.

President, could have been remembered for many things: perhaps for Poincaré's new intervention, "Du rôle de l'intuition et de la logique en Mathématiques"; maybe, although not likely, for Vito Volterra's lecture "Betti, Brioschi, Casorati, trois analystes italiens et trois manières d'envisager les questions d'analyse" or Gösta Mittag-Leffler's "Une page de la vie de Weierstrass". Maybe even because of some of the short communications, H. Padé's, "Aperçu sur les développements récents de la théorie des fractions continues", or Jacques Hadamard's "Sur les équations aux dérivées partielles à caractéristiques réelles". However, as it is well known, that congress is remembered, even now, more than a century afterwards, for David Hilbert's famous lecture "Mathematische Probleme." Nevertheless, Hilbert's presentation was not the opening lecture of the congress, such honour fell on the leading German historian of mathematics Moritz Cantor from Heidelberg, the author of the monumental *Vorlesungen über Geschichte der Mathematik* (*Lectures on the History of Mathematics*), the first volume of which appeared in 1880. Cantor's lecture (delivered in French) was entitled "Sur l'historiographie des Mathématiques", and was followed by the invited lecture of Volterra already mentioned. As a matter of fact, Hilbert's contribution was only a communication at 9 o'clock in the morning of August 8, followed by two more. The session should have been presided by prince Roland Bonaparte, but Napoléon's descendant did not appear – losing a unique occasion to be present, albeit in a secondary manner, in one of those rare occasions which could be called immortal, that his family had so much valued – and Moritz Cantor assumed the presidency. It is true that in the proceedings of the congress, Hilbert's intervention appeared in the part of "Conférences" (plenary lectures),[7] after Cantor's and Volterra's, but this was so because the editors recognised its importance once the text was received: "On trouvera plus loin," we can read in the proceedings, "le développement de la Communication de M. Hilbert qui, en raison de sa grande importance, a été placée parmi les conférences."[8] Hilbert was well recognised by his French colleagues, but only to give him the presidency of the August 7 and 9 sessions. Indeed, we know from a letter that Poincaré wrote to his good friend, the Swedish Gösta Mittag-Leffler on November 22, 1900, that by then Hilbert's text was not considered yet important or has not been received:[9] "On va mettre sous presse," Poincaré then wrote, "les *Comptes Rendus des travaux du congrès des Mathématicians*; on commençera naturellement par les conférences. Celles de MM. Cantor, Volterra et la mienne sont deja composées."

As to the closing session, it was occupied by the already mentioned lectures of Poincaré and Mittag-Leffler.

By the way, a Spaniard, Zoel García de Galdeano, from Zaragoza, presented a short (one page) communication, which was duly included in the proceedings: "Note

[7] David Hilbert, "Sur les problèmes futurs des mathématiques", *Compte Rendu du Deuxième Congrès International des Mathématiciens* (Gauthier-Villars, Paris 1902), pp. 58–114. In a footnote it was stated that "l'original de la traduction a paru en allemand dans les *Göttinger Nachrichten*, 1900. M. Hilbert a fait ici quelques modifications à l'original au § 13 et quelques additions au § 14 et au § 23".

[8] *Compte Rendu du Deuxième Congrès International des Mathématiciens*, *op. cit*., p. 24.

[9] *La correspondance entre Henri Poincaré et Gösta Mittag-Leffler*, Philippe Nabonnand, ed. (Birkhäuser, Basel 1990), p. 296.

sur la critique mathématique". He was one of the three Spaniards (among the 252 mathematicians) who attended the congress, the other two were Leonardo Torres Quevedo, the famous engineer, and José Rius y Casas, also from Zaragoza. Together with José Echegaray, García de Galdeano was the mathematician who did most to foster advanced mathematics in Spain during the XIXth century.

Among the many outstanding mathematicians participating in the meeting let us mention Émile Borel, Gaston Darboux, Ivar Fredholm, Jacques Hadamard, Tullio Levi-Civita, Hermann Minkowski, Paul Painlevé, Giuseppe Peano, Carl Runge and Giuseppe Veronese, as well as the physicists Joseph Larmor and Edmund Whittaker.

A final comment. Because of the historical importance of Hilbert's talk, we tend to assume that the Paris Congress must have been recognised by Parisians as a special occasion at the time. Not so. As a matter of fact, it was one of some 200 conferences held in Paris that year in connection with the World Exhibition.[10]

Heidelberg (1904), Klein's *Encyklopädie* and Ludwig Prandtl

I shall report on two events of the third congress. One is the presentation of Felix Klein's *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* (*Encyclopaedia of mathematical sciences, including its applications*), and the other is about one of the participants in the meeting, the applied mathematician and engineer Ludwig Prandtl, no one usually mentioned in the histories dealing with that congress. He was the world's leading expert in aerodynamics, a man who was able to move with equal grace between engineering and applied mathematics. He embodied one of Felix Klein's most dear ideals: the scholar who could promote at the same time both mathematics and an applied subject. In the words of Theodore von Kármán, another outstanding aerodynamics expert:[11] "Perhaps what one must wonder at most in Prandtl's scientific method, the direct connection of general, abstract theorems with experimental facts and practical applications, is pure, unadulterated Göttinger Tradition, which adapted by F. Klein in new form and to the demands of the technological century, has undergone a rejuvenation". It is therefore not surprising that Klein succeeded (in 1904) in getting Prandtl to Göttingen as professor and as one of the two directors (the other was Carl Runge) of a brand new Institute for Applied Mathematics and Mechanics.

That Third International Congress of Mathematicians took place in Heidelberg, organized by Heinrich Weber and was attended by 336 persons, i.e. 107 more than at the previous meeting. The largest delegation was from Germany (173), followed by Russia (30), Austria-Hungary (25), France (24), United States (15), Denmark (13), Italy (12) and Switzerland (12). Only one Spaniard (again García de Galdeano) participated.

[10] J. Gray, *The Hilbert Challenge*, *op. cit*., p. 59.

[11] Theodore von Kármán, "Ludwig Prandtl," *Zeitschrift für Flugtechnik und Motorluftschiffahrt* 16, 37 (1925).

It was there that the first volume of one of Klein's most ambitious mathematical enterprises, the monumental *Encyklopädie der mathematischen Wissenschaften*, which assembled all pure and applied mathematics (meaning not only "technological matters" but most physics branches) was presented. As a matter of fact, at the same time the first volume of the French version of the encyclopedia, the *Encyclopédie des sciences mathématiques*, was introduced. Its main editor, Jules Molk, from Nancy, was in charge of the presentation. As it is less known than its German counterpart, it might be of interest to quote some of Molk's words at the presentation:[12] "J'ai l'honneur de vous presenter le premier fascicule de l'édition française de l'Encyclopédie des sciences mathématiques. Cette édition française vient s'adjoindre à l'édition allemande de la même Encyclopédie. Ce n'est pas une simple traduction : c'est un exposé, fait par des mathematiciens de langue française, des articles contenus dans l'édition allemande ; ces articles son complétés, mis à jour ; le mode d'exposition est d'ailleurs entièrement conforme aux traditions françaises".

It was at Heidelberg, too, that Ludwig Prandtl presented (as one of the eighty papers read) his most admired and enduring scientific contribution: the boundary-layer hypothesis (that a state of flow can be approximated by a wall zone of viscous influence and an outer zone of irrotational motion), which whould later prove essential in the then small core of fundamental concepts constituting the theory of aerodynamics.[13] Apparently, however, it "received only passing attention from the mathematicians who heard it", according to the historians of hydraulics, Hunter Rouse and Simon Ince. Its significance, of course, was not lost on Klein.[14]

I mentioned before Cantor's *Vorlesungen über Geschichte der Mathematik* and said that the first volume had been published in 1880. The second appeared in 1893, and the third in several parts between 1894 and 1898. These volumes covered the history of mathematics from its beginnings up to 1758. It was at the Heidelberg Congress that a fourth volume was planned that would go up to 1799. This volume was published in 1908, written by nine historians of mathematics, with Cantor as editor-in-chief. We may therefore say that the International Congresses of Mathematicians also form part of the historiography of mathematics, or, to express it differently, to the history of the history of mathematics.

As to the general lectures in Heidelberg, they were delivered by: L. Könisgsberger ("Carl Gustav Jacob Jacobi"), P. Painlevé ("Le problème moderne de l'intégration des équations différentielles"), A. G. Greenhill ("The Mathematical Theory of the Top considered historically"), C. Segre ("La Geometria d'oggidì e i suoi legami coll' Analisi"), and W. Wirtinger ("Riemanns Vorlesungen über die hypergeometrische Reihe und ihre Bedeutung").

[12] *Verhandlungen des Dritten Internationalen Mathematiker-Kongresses in Heidelberg vom 8.bis 13. August 1904*, A. Krazer, ed. (Druck und Verlag von B. G. Teubner, Leipzig 1905), p. 36.

[13] Ludwig Prandtl, "Über Flüssigkeitsbewegung bei sehr kleiner Reibung," *Verhandlungen des Dritten Internationalen Mathematiker-Kongresses*, pp. 484–491.

[14] Hunter Rouse and Simon Ince, *History of Hydraulics* (Institute of Hydraulic Research, Iowa City 1957), p. 230; quoted in Paul A. Hanle, *Bringing Aerodynamics to America* (The MIT Press, Cambridge, Mass., 1982), p. 43.

Rome (1908), Volterra's reign

The Fourth International Congress convened in the spring of 1908 in Rome, a fitting tribute to the Italian mathematics eminence. Departing from the tradition of previous congresses, the Italians doubled the number of plenary addresses. Hilbert and Klein were invited but they had to decline. Darboux and Veronese were among the speakers. So was Poincaré, but, again, he felt ill and could not deliver his lecture (it was read by Gaston Darboux, who also was in charge of one of the general lectures, "Les origines, les méthodes et les problèmes de la Géométrie infinitésimale").[15] It was the last time that Poincaré was to attend one of these meetings: he died a few weeks before the next one was inaugurated.

The opening address was entrusted to Vito Volterra, who spoke about "Le Matematiche in Italia nella seconda metà del Secolo XIX."[16] The other main speakers were Walther von Dyck, Andrew R. Forsyth, Gösta Mittag-Leffler, the American astronomer Simon Newcomb, Émile Picard, Giuseppe Veronese, and the physicist Hendrik A. Lorentz, who tackled a fundamental problem which would play a central role in the origins of quantum physics: "Le partage de l'énergie entre la matière pondérable et l'éther." Almost nine years after Max Planck had introduced his radiation law as well as the quanta, Lorentz told his Rome audience how uncertain the situation still was:[17] "… si l'on compare la théorie de Planck et celle de Jeans, on trouve qu'elles ont toutes les deux leurs mérites et leurs défauts. La théorie de Planck est la seule qui nous ait donné une formule conforme aux résultats des expériences, mais nous ne pouvons l'adopter qu'à condition de remanier profondément nos idées fondamentales sur les phénomènes électromagnétiques… La théorie de Jeans, au contraire, nous oblige à attribuer à un hasard pour le moment inexplicable l'accord entre les observations et les lois de Boltzmann et de Wien."

Almost as many of the papers were presented in French (51) as in Italian (53). Max Noether, the noted algebraist, brought his daughter Emmy as a guest. I must confess that I have a special predilection for Emmy Noether. Not only because of all the difficulties she suffered during her life – just because she was a woman, a female mathematician in a men's world – but also for her "Noether's theorems", which relate symmetries to conservations laws. Despite of Hilbert, Klein and Hermann Weyl's support – and of Einstein's – Noether never could get an official, not to say permanent, position in Göttingen. When in January 1933 Hitler assumed the power in Germany, she had to abandon her country, and travelled to the United States, to the Mathematics

[15] H. Poincaré, "L'avenir des mathématiques", *Atti del IV^e Congresso Internazionale dei Matematici*, G. Castelnuovo, ed. (Tipografia della R. Accademia dei Lincei, Roma 1909), vol. I, pp. 167–182; also published in *Rendiconti del Circolo Matematico di Palermo* 26, 152–168 (1908).

[16] During the first congresses, history of mathematics appeared rather frequently among the topics considered. Besides Volterra's lecture, in Rome Gino Loria launched a project for the production of a textbook for training in the history of mathematics. When the project failed because of World War I, Loria published material he had collected as a *Guide allo studio della storia della matematica* (1916; second expanded edition, 1946).

[17] H. A. Lorentz, "Le partage de l'énergie entre la matière pondérable et l'éther", *Atti del IV^e Congresso Internazionale dei Matematici*, op. cit., vol. I, pp. 145–165; p. 163; reprinted in H. A. Lorentz, *Collected Papers*, vol. VII (Martinus Nijhoff, The Hague 1934), pp. 316–342; p. 341.

Department at Bryn Mawr College, a women's college that offered her the hope of a new life. Alas, she died soon, as a consequence of a surgical operation she underwent in April 1935. She was only 52 years old.

Cambridge, 1912, the heritage of the Mathematical Tripos

The Fifth International Congress took place at Cambridge in Great Britain (August 22–28, 1912). It was attended by 574 mathematicians. Lord Rayleigh was the Honorary President and Sir George H. Darwin the effective President, as president of the Cambridge Philosophical Society, which was the institution responsible for the organization of the event. A physicist the first and a geophysicist the second, both were products of the Mathematical Tripos, the educational system that reigned supreme in Cambridge since the eighteenth century.[18] Unfortunately, I have no time here to explain this system that favoured more the development of mathematical physics than of mathematics in Britain; I will only say that its emphasis was on applied mathematics, and that many of the most distinguished British physicists, as well, of course, of mathematicians, were products of the Tripos, among them George Gabriel Stokes (1841)[19], Arthur Cayley (1842), William Thomson (Lord Kelvin) (1845), Peter Tait (1852), Edward Routh (1854), James Clerk Maxwell (1854), John Henry Poynting (1876), Joseph Larmor (1880), Joseph J. Thomson (1880), Bertrand Russell (1893), Edmund Whittaker (1895), James Jeans (1898), Godfrey Harold Hardy (1898), James Chadwick (1900), Arthur Eddington (1904) and John Edensor Littlewood (1905).[20] In general, physicists did better than mathematicians in the Tripos, due to its applied character.

Probably in no other congress, before or afterwards, pure and applied mathematics mixed up more. If we look, for instance, at the plenary lectures, we find that of the eight delivered, half of them were dedicated to applied mathematics: Ernest W. Brown ("Periodicities in the solar system"), Prince B. Galitzin ("The principles of instrumental seismology"), Joseph Larmor ("The Dynamics of Radiation"), and W. H. White ("The place of Mathematics in Engineering Practice"). The remaining four were given by Maxime Bôcher, Émile Borel, Federigo Enriques and Edmund Landau, all "pure" mathematicians. As to the sections, they were dedicated to: "Arithmetic, Algebra, Analysis," "Geometry," "Mechanics, Physical Mathematics, Astronomy" (with 25 communications presented by scientists like Max Abraham, Peter Paul Ewald, Horace Lamb, A. E. H. Love, Ludwig Silberstein, Marian von Smoluchowski, J. J. Thomson, and the Spanish Esteban Terradas, who spoke, in French, "Sur le mouvement d'un fil"), and, finally, the sections "Economics, Actuarial Science, Statistics," and "Philosophy, History, Didactics."

[18] Rayleigh (J. W. Strutt) was *senior wrangler* in 1865, and George Darwin *second wrangler* in 1868.

[19] The year in brackets is the year of graduation.

[20] See Andrew Warwick, *Masters of Theory. Cambridge and the Rise of Mathematical Physics* (The University of Chicago Press, Chicago 2003).

As I said before, Poincaré had died when the congress began. At the opening session, G. H. Darwin remembered him with the following words:[21]

> "Up to a few weeks ago there was one man who alone of all mathematicians might have occupied the place which I hold without misgivings as to his fitness; I mean Henri Poincaré. It was at Rome just four years ago that the first dark shadow fell on us of that illness which has now terminated so fatally. You all remember the dismay which fell on us when the word was passed from man to man 'Poincaré is ill.' We had hoped that we might again have heard from his mouth some such luminous address as that which he gave at Rome; but it was not to be, and the loss of France in his death affects the whole world."

Before remembering Poincaré, Darwin had said something which I also wish to quote:[22] "The Science of Mathematics is now so wide and is already so much specialised that it may be doubted whether there exists to-day any man fully competent to understand mathematical research in all its many diverse branches. I, at least, feel how profoundly ill-equipped I am to represent our Society as regards all that vast field of knowledge which we classify as pure mathematics. I must tell you frankly that when I gaze on some of the papers written by men in this room I feel myself much in the same position as if they were written in Sanskrit".

Perhaps these words of a man like George Darwin, a *second wrangler* in the Tripos, by no means a mathematical ignorant, might console some – or many – of us when we look now upon the titles of the present congress.

Immediately after his confession, Darwin went on to proudly add:

> "But if there is any place in the world in which so one-sided a President of the body which has the honour to bid you welcome is not wholly out of place it is perhaps Cambridge. It is true that there have been in the past at Cambridge great pure mathematicians such as Cayley and Sylvester, but we surely may claim without undue boasting that our University has played a conspicuous part in the advance of applied mathematics. Newton was a glory of all mankind, yet we Cambridge men are proud that fate ordained that he should have been Lucasian Professor here. But as regards the part played by Cambridge I refer rather to the men of the last hundred years, such as Airy, Adams, Maxwell, Stokes, Kelvin, and other lesser lights, who have marked out the lines of research in applied mathematics as studied in this University. Then too there are others such as our Chancellor, Lord Rayleigh, who are happily still with us."

[21] *Proceedings of the Fifth International Congress of Mathematicians* (*Cambridge, 22–28 August 1912*), E. W. Hobson and A. E. H. Love, eds. (Cambridge University Press, Cambridge 1913), vol. I, p. 34.

[22] Ibid., pp. 33–34.

Mathematics and politics: Strasbourg (1920), Toronto (1924), Bologna (1928)

Two years after the Cambridge Congress, the Great War, later called World War I, broke out, and the mathematical congresses suffered from that. Not only there were no meetings until 1920, but the consequences of the war affected the mathematical community for years.

The 1920 congress should have taken place in Stockholm, the site elected in Cambridge for the 1916 meeting, but politics interfered and Strasbourg, which had been annexed by the Germans after the Franco–Prussian war of 1870–1871 and reclaimed by the French at the Treaty of Versailles along with the rest of Alsace and Lorraine, took its place. It was, of course, a demonstration of political power – and revenge – by the victors of the war. Moreover, German mathematicians were not invited. Some, as Hardy, Littlewood or Mittag-Leffler objected, but to no avail. In his address for the closing ceremony, Émile Picard, who presided with Camille Jordan as Honorary President, stated:[23]

> "En ce qui regarde spécialement notre Congrès, nous n'avons jamais dissimulé que nous entendions lui donner une signification particulière, en le réunissant à Strasbourg. Aussi avons-nous été extrêmement touchés de l'empressement avec lequel nos amis étrangers ont répondu à notre appel… Des liens plus intimes ont été formés, qui resteront précieux. Nous continuerons ainsi, entre peuples amis, nos travaux scientifiques, apportant dans cette collaboration nos qualités diverses, sans qu'aucun prétende exercer une insupportable hégémonie et sans nous soucier de certaines menaces, qu'avec une impudeur qui ne nous étonne pas, on a osé proférer.
>
> Quant à certaines relations, qui ont été rompues par la tragédie de ces dernières années, nos successeurs verront si un temps suffisamment long et un repentir sincère pourront permettre de les reprendre un jour, et si ceux qui se sont exclus du concert des nations civilisées sont dignes d'y rentrer. Pour nous, trop proches des événements, nous faisons encore nôtre la belle parole prononcée pendant la guerre par le cardinal Mercier, que, pardonner à certains crimes, c'est s'en faire le complice."

Mathematicians from 27 countries attended the congress, which was held from September 22 to September 30. Not surprising, the great majority (82 per cent) of the talks were in French and just 17 per cent were given in English. Only one speaker – Rudolf Fueter, from Zurich – lectured in German. One of the American participants of the congress was Norbert Wiener, who attended as MIT's representative and presented a paper on Brownian motion. It was his first International Congress of

[23] *Comptes Rendus du Congrès International des Mathématiciens. Strasbourg, 1920*, Henry Villat, ed. (Imprimerie et Librairie Privat, Toulouse 1921), pp. xxxi–xxxiii; quoted in Angelo Guerraggio and Pietro Nastasi, *Italian Mathematics Between the Two World Wars* (Birkhäuser, Basel 2005), pp. 57–58.

Mathematicians.

Toronto 1924, followed (it should have been New York, but in April 1922 the Americans withdrew – for reasons not stated in their *Bulletin* – in favour of the Canadian city). Again, no Germans were present. The plenary lecturers were Leonard Dickson, Joseph Larmor, Niels Nörlund, Charles de la Vallée-Poussin, and Vito Volterra.

A German delegation of 67 mathematicians, headed by Hilbert, attended the next congress, Bologna 1928, ten years after the end of the war. Not all the German mathematicians were in favour of that decision. One group opposed it. Its leader was Ludwig Bieberbach, who when Hitler assumed power in 1933 propagated a *Deutsche Mathematik*; that is, a mathematics free from the "Jewish spirit," whatever that might had meant, and, of course, from mathematicians of Jewish origin. There is, however, a fact we must not forget: Italy was then Mussolini's Italy, the future ally of Hitler's Germany. As a matter of fact, when we look over the *Atti* (proceedings) of the congress, we find that the *Comitato d'onore* was presided by "S. E. Cav. Benito Mussolini, Capo del Governo". Among the other members of the committee were T. Tittoni, *Presidente del Senato*, A. Casertano, *Presidente della Camera dei Deputati*, A. Turati, *Segretario del Partito Fascista*, and P. Badoglio, *Maresciallo d'Italia, Capo di Stato Maggiore Generale*.

Constance Reid, Hilbert's biographer, described what happened during the opening ceremony. I will use her words:[24]

> "At the opening session, as the Germans came into an international meeting for the first time since the war, the delegates saw a familiar figure, more frail than they remembered, marching at their head. For a few minutes there was not a sound in the hall. Then, spontaneously, every person present rose and applauded.
>
> 'It makes me very happy,' Hilbert told them in the familiar accent, 'that after a long, hard time all the mathematicians of the world are represented here. This is as it should be and as it must be for the prosperity of our beloved science.
>
> 'Let us consider that we as mathematicians stand on the highest pinnacle of the cultivation of the exact sciences. We have no other choice than to assume this highest place, because all limits, especially national ones, are contrary to the nature of mathematics. It is a complete misunderstanding of our science to construct differences according to peoples and races, and the reasons for which this has been done are very shabby ones.
>
> 'Mathematics knows no races… For mathematics, the whole cultural world is a single country.' "

This is the sort of words we must remember, the jewels of history. They are as valid today as they were when Hilbert pronounced them, and we can very easily understand the emotion they aroused in the people that heard them.

[24] Constance Reid, *Hilbert* (Springer-Verlag, Berlin 1970), p. 188.

In the paper he read in Bologna, Hilbert considered a very important problem, the so-called *Entscheidungsproblem*, or "decision problem": to show whether there exists an algorithm for deciding if a mathematical proposition is a logical consequence of others. He had already considered it in his 1900 Paris lecture as part of problem number 2 ("the compatibility of the arithmetic axioms"). This line of research would culminate in the works of Alonzo Church, Alan Turing and Kurt Gödel.

Besides Hilbert's talk, plenary lectures were delivered also by: Luigi Amoroso ("Le equazioni differenziali della dinamica economica"), George D. Birkhoff ("Quelques éléments mathématiques de l'art"), Émile Borel ("Le Calcul des probabilités et les sciences exactes"), Guido Castelnuovo ("La geometria algebrica e la scuola italiana"), Maurice Fréchet ("L'analyse générale et les Espaces Abstraits"), Jacques Hadamard ("Le développement et le rôle scientifique du Calcul fonctionnel"), Theodore von Kármán ("Mathematische Probleme der modernen Aerodynamik"), Nikolai Lusin ("Sur les voies de la théorie des ensembles"), Roberto Marcolongo ("Leonardo da Vinci nella storia della matematica e della meccanica"), Umberto Puppini ("Le boniche in Italia"), Leonida Tonelli ("Il contributo italiano alla teoria delle funzioni di variabili reali"), Oswald Veblen ("Differential Invariants and Geometry"), Vito Volterra ("La teoria dei funzionali applicata ai fenomeni ereditari"), Hermann Weyl ("Kontinuierliche Gruppen und ihre Darstellungen durch lineare Transformationen") and William Young ("The mathematical method and its limitations").

Zurich (1932): A woman plenary lecturer

I mentioned before Emmy Noether and the problems she had because of being a woman. The International Congresses of Mathematicians treated her better than the authorities of her country; in particular the 1932 International Congress of Mathematicians, held in September in Zurich again (there will be a third time, in 1994). She delivered one of the plenary lectures, the first woman to achieve such honour in the history of the congresses.[25] On September 7, she spoke on "Hyperkomplexe Systeme in ihren Beziehungen zur kommutativen Algebra und zur Zahlentheorie" ("Hypercomplex systems in their relations to commutative algebra and to number theory").[26]

[25] It is interesting what D. J. Albers, G. L. Alexanderson and C. Reid (*International Mathematical Congresses. An Illustrated History, 1893–1986*, *op. cit*., p. 23) wrote concerning the presence of women at the congresses of mathematicians: "Since the World Congress held in Chicago, women mathematicians had been in attendance at all but one of the international congresses. At Zurich in 1897, however, they were listed under 'Damen,' which classification consisted otherwise of the wives and daughters of the attending mathematicians. Attendance of women mathematicians through the second Zurich congress was as follows: Chicago, 3; Zurich, 4; Paris, 6; Heidelberg, 0; Rome, 19; Cambridge, 37; Strasbourg, 6; Toronto, 25; Bologna, 69; and Zurich, 35 (total congress attendance having declined)."

[26] As a proof that tensions were not absent in the mathematical community, we have that a year later, and referring to this talk, George Birkhoff accused Noether of intentional ignorance of Anglo-American scientists. Maria Georgiadou, *Constantin Carathéodory. Mathematics and Politics in Turbulent Times* (Springer-Verlag, Berlin 2004), p. 272.

247 official delegates and 420 participants attended this congress. It was presided by Karl Rudolf Fueter, professor at the Zurich University, which, together with the Polytechnic (or Federal Institute of Technology), sponsored the meeting. After Fueter's opening speech, Constantin Carathéodory delivered the lecture "Über die analytischen Abbildungen von mehrdimensionalen Räumen" ("On analytic mappings of multidimensional spaces").

Together with Noether and Carathéodory, and among others, plenary lecturers were: James Alexander (who spoke about "Some problems in topology"), Ludwig Bieberbach ("Operationsbereiche von Funktionen"), Harald Bohr ("Fastperiodische Funktionen einer komplexen Veränderlichen"), Élie Cartan ("Les espaces riemanniens symétriques"), Gaston Julia ("Essai sur le développement de la théorie des fonctions de variables complexes"), Karl Menger ("Neuere Methoden und Probleme der Geometrie"), Marston Morse ("The calculus of variations in the large"), as well as the physicist Wolfgang Pauli, professor at Zurich, who spoke about the "Mathematische Methoden der Quantenmechanik." Also present, although no plenary lecturers, were Edmund Landau, Hermann Weyl, Richard Courant, and the splendid mathematician and unforgettable author of *A Mathematician's Apology* (1940), Godfrey Hardy. By the way, according to George Pólya, Landau claimed that he went to Zurich to play bridge with Hardy. I do not know if they played, but Landau must have been a difficult adversary: he was a very rich person, a millionaire.[27]

It was in Zurich 1932 that it was announced a bequest from the will of the Canadian John Charles Fields (1863–1932), who as a mathematician had cultivated the theory of algebraic functions and algebra. Fields died on August 8, 1932, just one month before the congress opened, but he had been working on the establishment of a prize for mathematicians. His friend, the mathematician and physicist John Lighton Synge (who had worked with Fields on the organization of the 1924 Toronto Congress) presented the proposal to the executive committee of the Zurich Congress. According to Michael Monastyrsky, the "session of the ad hoc committee that considered the question of the prize was stormy. Not all the members of the committee supported the establishment of a prize. In particular Oswald Veblen spoke against it, perhaps motivated by the thesis that the study of science is its own reward, so the researcher has no need of additional encouragement. Nevertheless, most of the committee members favoured Fields' proposal. At the plenary session of the congress, the question was finally decided in the affirmative."[28]

Oslo (1936). The first Fields medals

The next congress was held in Oslo. It can be remembered mainly for two facts. The first is that since 1933 Germany was already Hitler's Germany, and some of the

[27] George Pólya, *The Pólya Picture Album. Encounters of a Mathematician* (Birkhäuser, Basel 1987), p. 88.

[28] M. Monastyrsky, *Modern Mathematics in the Light of the Fields Medals* (AK Peters, Wellesley, Mass. 1988), p. 7.

best German mathematicians had fled their native country. Hermann Weyl, Richard Courant, Felix Bernstein, and Emmy Noether were no longer in Göttingen, nor were the young *Privatdozenten* Paul Bernays, Hans Lewy and Otto Neugebauer, who lost their *venia legendi*. Also Richard von Mises (he went first to Turkey, and afterwards to the United States) and Kurt Gödel left Germany. Edmund Landau had lost his chair, although he did not live to see the worst (he died in 1938). The cases of Felix Hausdorff and Otto Blumenthal were worse. Hausdorff stayed in Germany, but in January 1942, when he learnt that he was going to be interned in a concentration camp, he committed suicide. Blumenthal left Germany for Holland, but when this country was taken by the Germans, he was interned and died in a camp.

Italian mathematics also suffered. Between July and November 1938, Benito Mussolini introduced racial laws that led to the removal of many mathematicians from their university positions, because of their Jewish origins. This concerned, among others, Guido Ascoli, Federigo Enriques, Gino Fano, Beppo Levi and Tullio Levi-Civita. Most of them had attended the Bologna congress in 1928 presided by the *Duce*.

The second event at the Oslo Congress that deserves to be remembered is that it was there that the first Fields medals were awarded. They were given to Lars Ahlfors (then 29 years old) and to Jesse Douglas (39), for their work in complex analysis and the solution of the Plateau's problem (i.e., the theory of minimal surfaces), respectively.[29] By the way, Ahlfors only learnt that he was the winner on the eve of the ceremony and was officially notified one hour before the congress opened. Such secrecy apparently resulted in Douglas, the other winner, not coming to the congress at all.[30]

In Oslo, it was decided that the next congress would be celebrated in America, at Harvard University, Cambridge, Massachusetts, in 1940, organised by the American Mathematical Society. Still in June 1939, advertisements appeared in journals announcing its celebration, due to be from September 4th to 12th. The provisional program included sections on: I. Algebra and number theory; II. Analysis; III. Geometry; IV. Probability calculus and economy; V. Mathematical physics and applied mathematics; VI. Logic, philosophy, history and didactics.[31] It took place there, but ten years later, in 1950.

Cambridge, Mass. (1950), the Fields medal to Laurent Schwartz and the road toward mathematical purity

Indeed, the first post-war International Congress of Mathematicians was held in 1950 in the US Cambridge (Massachusetts). Many things had changed since the times of the mathematics congress associated with the Chicago 1893 World's Fair: no longer,

[29] J. Plateau was a Belgian physicist, who in 1847 performed a series of experiments with soap films that led to the creation of the theory of minimal surfaces.

[30] Michael Monastyrsky, *Modern Mathematics in the Light of the Fields Medals*, *op. cit*., p. 16.

[31] I have consulted an announcement that appeared in *La Ricerca Scientifica* (June 1939), pp. 604–605.

for instance, were American mathematicians so dependant on European mathematics as it had been then.

The proceedings of the Cambridge congress provide us with a rather good idea of the new and now enlarged space occupied by the discipline. Mathematicians whose wartime activities had enriched their disciplines – such as John von Neumann (theory of shock waves), Norbert Wiener (theory of statistics of prediction), Claude Shannon (information theory) and Abraham Wald (statistics) – were invited to present their latest results alongside speakers on pure mathematics – Oscar Zariski (algebraic varieties), André Weil (algebraic geometry), Henri Cartan (theory of analytic manifolds), Withold Hurewitz (homotopy), Claude Chevalley (algebraic groups) – whose work belonged to the tradition of the most prestigious mathematical sub-disciplines. It was, probably, the last occasion in which that happened. In the next congresses, "a clean 'take-over of power' by structural Hilbertian mathematicians", using Amy Dahan Dalmedico's words,[32] took place. The number of sessions on, for example, algebra, algebraic geometry, and algebraic topology increased from congress to congress, while the number of sessions dedicated to mathematical physics, statistics and applied branches fell regularly. Large parts of classical analysis and differential equations were going to be considered for some time as more or less exhausted of interest. The interaction between mathematics and the social sciences almost disappeared from international congresses, at least until more recent congresses. Of the 21 Fields medals awarded between 1950 and 1978, four were for number theory – Atle Selberg (1950), Klaus Roth (1958), Alan Baker (1970), Enrico Bombieri (1974) – seven for algebraic geometry, five for differential topology and algebra, while only three were for analysis (Laurent Schwartz (1950), Lars Hörmander (1962), Charles Fefferman (1978)), and there were none for probability theory. The domain associated with partial differential equations was only given the medal in 1994 (Jean-Christophe Yoccoz). I think that this trend has begun to change a bit recently, with the new development of applied mathematics helped by computers, but it is too early to judge. Perhaps significant in this context is the Fields medal awarded in 1990 to Edward Witten, a man formally educated as a physicist and interested essentially in physics problems (string theories, mainly). "In honouring him," wrote Monastyrsky, "the mathematical community was recognizing the exceptional importance of the penetration of physical ideas and methods into modern mathematics. Recent papers of medallists Jones and Drinfel'd also concern to a degree mathematical physics, or, from a different point of view, physical mathematics."[33]

The Fields medals are now well known throughout the world. This was not so, however, in the beginning, a development which, in accord with the title of this article, reflects the road from the private to the public. The case of Laurent Schwartz, the analyst who received the prize for his work in the theory of generalized functions (distributions), is a good example of this.

[32] Amy Dahan Dalmedico, "Mathematics in the twentieth century", in John Krige and Dominique Pestre, eds., *Science in the Twentieth Century* (Harwood Academic Publishers, Amsterdam 1997), pp. 651–667; p. 659.

[33] M. Monastyrsky, *Modern Mathematics in the Light of the Fields Medals*, *op. cit*., p. 105.

Schwartz received the Fields medal at the 1950 Cambridge congress (the other Fields medallist was Atle Selberg, honoured for his work in number theory, more specifically on the zeros of the Riemann zeta function and for developing an extraordinarily efficient method of estimating the distribution of primes). It is interesting to recall that Schwartz had problems to enter the United States: he was a former Trotskyst and decided leftist, and so he was denied a visa. It was thanks to the efforts of the American Mathematical Society, which pleaded his cause with president Truman, so that he got his visa some months before the meeting. Jacques Hadamard, then 85 years old, who was not a Communist but had visited the Soviet Union several times and who was one of the Honorary Presidents, was also denied a visa. His case was solved only five days before the opening of the congress.

It was the second time that the medals were awarded, since, as I said, there had been no congresses since 1936, due to World War II. In his autobiography, Schwartz remembered the events connected with his medal.[34] "During the summer of 1949, Marie-Hélène [his wife] sent me to Canada a letter that informed me that I will receive the Fields medal at the next International Congress of Mathematicians, that was to be held in the summer of 1950. I did not know at all what that was [*j'ignorais absolutement ce que c'était*]. I was to receive a gold medal, together with 500 Canadian dollars (six thousand francs), given to two mathematicians every four years, during an international congress, for my works on the distributions. The letter gave me all the necessary explications, pointing out everything would be confidential until the day of the Cambridge, Massachusetts, Congress". And Schwartz added: "The recompense was equivalent to less than two months of my monthly salary".

Thus, we have that this now very much valued medals, which today receive world attention when they are presented, were a much humble award at first. A long road has been covered to go from the private to the public.

Following the road: from Amsterdam (1954) to Madrid (2006)

After Cambridge, other congresses came, always with an increasing number of audiences. Amsterdam (1954), Edinburgh (1958), Stockholm (1962), Moscow (1966), Nice (1970), Vancouver (1974), Helsinki (1978), Warsaw (1983; the one-year delay was due to political reasons), Berkeley (1986), Kyoto (1990), Zurich (1994), Berlin (1998), Beijing (2002), and the last one, for the time being, Madrid (2006). It would be not just difficult but certainly impossible to cover all of them in a single lecture. In Edinburgh, 1958, for instance, the number of one-hour lectures by invitation of the organizing committee was already 19, with speakers such as Aleksandr Aleksandrov, Aleksei Bogolyubov, Henri Cartan, Claude Chevalley, Cornelius Lancos, Lev Pontryagyn, Stephen Kleene, René Thom, George Temple, and the physicist George Uhlenbeck.[35]

[34] Laurent Schwartz, *Un mathématicien aux prises avec le siècle* (Edtions Odile Jacob, Paris 1997), pp. 319–321.

Perhaps I should remember the Moscow 1966 Congress. More than 4 300 attended it from 54 countries, including delegations from Cuba, North Korea and North Vietnam, and more than a dozen others from Latin American, African and Asian countries. Of course, politics melt there quite clearly with mathematics, but anyhow it was an important occasion. A special stamp was issued to commemorate the Congress, the same that happened in 1978, when Finland issued another to mark the Helsinki Congress, and as has again happened in Spain 2006. And what is more public than a stamp? Another demonstration of the road that has taken the international community of mathematicians from the private to the public.

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[35] See *Proceedings of the International Congress of Mathematicians, 14–21 August 1958*, J. A. Todd, ed. (Cambridge University Press, Cambridge 1960), p. xiv.