Competitions and mathematics education

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Abstract. Mathematics competitions, together with the people and organizations engaged with them, form an immense and vibrant global network today. This network has many roles. Competitions help identify students with higher abilities in mathematics. They motivate these students to develop their talents and to seek professional realization in science. Competitions have positive impact on education and on educational institutions. Last but not least, a significant part of the classical mathematical heritage known as "Elementary Mathematics" is preserved, kept alive and developed through the network of competitions and competition-related activities. Nevertheless, competitions need to evolve in order to meet the demands of the new century.

These and many other items are outlined and discussed in the paper.

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1. Introduction

Competition is essential and intrinsic to life. Every day, living things in nature and economic subjects in society compete for resources, for better living conditions, and for higher efficiency. The desire to compete in overcoming a challenge is deeply rooted in human nature and has been employed for centuries to help people sharpen their skills and improve their performance in various activities.

Competitions, however hotly debated, praised, or condemned, remain central and inherent in education. Both the traditional marking (grading) of students in school and the more innovative measuring of their basic scholastic abilities (implemented by methods such as PISA, TIMSS, or SAT) inevitably create, directly or indirectly, competition among students, among teachers, among schools, and even among whole countries. Heated debates aside, few would deny the positive influence such competitions bring to the process of teaching and learning, and to the overall performance of the educational system.

The interaction between competition and education is more complex, however. It is not only that competitions enhance education. Education itself can be viewed as

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preparation of individuals (or groups of individuals, even whole nations) for future competitions.

In what follows, we give a brief history of contemporary math competitions and present the state of the art in this area. Then we outline how competitions help identify, motivate, and develop higher-ability and talented students. Next we focus on the impact of competitions on education, on educational institutions and on mathematics as a science. Finally, we pose challenges and identify venues for improvement.

2. Brief history of mathematics competitions

It is difficult to trace precisely the origins of mathematics competitions for school students; after all, in-class testing (which often resembles small-scale competitions) has accompanied the school system from its very beginning. In fact, the archetype of some competitions can be found outside school, in the society. Newspapers and recreational journals frequently offer prizes for solving crosswords, puzzles, and problems of a deeper mathematical nature. This practice is widely used today by many mathematical journals that publish problems and give awards to school students who provide good solutions.

V. Berinde [2] reports that a primary school math competition with 70 participants was held in Bucharest, Romania, as early as 1885. There were eleven prizes awarded to 2 girls and 9 boys. It cannot be excluded that other competitions were held elsewhere before or after that date too. Nevertheless, the 1894 Eötvös competition in Hungary is widely credited as the forerunner of contemporary mathematics (and physics) competitions for secondary school students. The competitors were given four hours to solve three problems individually (no interaction with other students or teachers was allowed). The problems in the Eötvös competition were specially designed to challenge and check creativity and mathematical thinking, not just acquired technical skills; the students were often asked to prove a statement.

As an illustration, here are the three problems given in the very first Eötvös competition in 1894 (the entire collection of problems and their solutions is maintained by John Scholes at www.kalva.demon.co.uk/eotvos.html):

P1. Show that $\{(m,n) : 17 \text{ divides } 2m+3n\} = \{(m,n) : 17 \text{ divides } 9m+5n\}$.

P2. Given a circle C, and two points A, B inside it, construct a right-angled triangle PQR with vertices on C and hypotenuse QR such that A lies on the side PQ and B lies on the side PR. For which A, B is this not possible?

P3. A triangle has sides length a, $a+d$, $a+2d$ and area S. Find its sides and angles in terms of d and S. Give numerical answers for $d=1$, $S=6$.

The Eötvös competition model still dominates the competition scene.

The year 1894 is notable also for the birth of the famous mathematics journal *KöMaL* (an acronym of the Hungarian name of the journal, which translates to *High School Mathematics and Physics Journal*). Founded by Dániel Arany, a high school teacher in Györ, Hungary, the journal was essential to the preparation of students and teachers for competitions (about one third of each issue was devoted to problems and problem solving and readers were asked to send solutions). As noted by G. Berzsenyi in the preface of [3], about 120–150 problems were published in *KöMaL* each year; about 2500–3000 solutions were received. The best solutions and the names of their authors were published in following issues. This type of year-round competition helped many young people discover and develop their mathematical abilities; many of them later became world-famous scientists. (For more information, see the journal web site, komal.elte.hu.)

About the same time, similar development occurred in Hungary's neighbor, Romania. The first issue of the monthly *Gazeta Matematică*, an important journal for Romanian mathematics, was published in September 1895. The journal organized a competition for school students, which improved in format over the years and eventually gave birth to The National Mathematical Olympiad in Romania. For legal reasons, the journal was transformed to *Society Gazeta Matematică* in August 1909. The following year, the Romanian Parliament approved the legal status of the new society and this is considered to be the birthday of the Romanian Mathematical Society [2].

What happened in Hungary and Romania in the late 1800's was not something isolated and special to these two countries only; most likely, it reflected a much broader trend. Indeed, international collaboration and solidarity were rising steadily and many national math societies were founded around the same time. The Olympic Games were revived in 1896. The First International Congress of Mathematicians took place in Zürich in 1897. Within several decades, other countries started to organize mathematics competitions. In 1934, a Mathematical Olympiad was organized in Leningrad, USSR (now St. Petersburg, Russia).

3. Mathematics competitions today

Today the world of mathematics competitions encompasses millions of students, teachers, research mathematicians, educational authorities, and parents, who organize and take part in hundreds of competitions and competition-like events with national, regional, and international importance every year. Even greater is the number of books, journals, and other printed and electronic resources that help students and their mentors prepare for the various types of competitions.

3.1. International Mathematical Olympiad (IMO). Of course, the most important and most prestigious math competition is the *International Mathematical Olympiad* (IMO) – an annual competition for high school students. Directly or indirectly, all other competition activities in mathematics and sciences are related to the IMO.

The idea to organize an international mathematics competition crystallized during the Fourth Congress of Romanian Mathematicians in 1956. Paul Jainta [4] points out that "IMO, the pinnacle of competitions among individuals, was the brainchild of Romania's Tiberiu Roman, an educator of monumental vision." The first IMO took place in Romania (1959) with participants from seven countries: Bulgaria, Czechoslovakia, German Democratic Republic, Hungary, Poland, Romania, and the Soviet Union (USSR). The second IMO (1960) was organized by Romania as well, but since then it is hosted by a different country every year (except 1980, when no IMO was held). Over the years, the participation grew dramatically: the 2005 IMO in Mexico gathered 513 competitors from 93 countries!

Strict formalized rules govern every aspect of the IMO, such as participation, problem selection, assessment of solutions, and distribution of medals (for a description of the IMO, browse erdos.fciencias.unam.mx).

Each country sends a team of up to eight (four in 1982; since 1983, six) high-school students, chaperoned by a team leader and a deputy team leader. The competition itself is held on two consecutive days; each day, the students have four and a half hours to solve three problems. Each year, just before the competition, the six problems are selected by an international jury formed by the national team leaders and representatives of the host country. Even though confined to secondary school mathematics, the problems are rather difficult and solving them requires a significant degree of inventive ingenuity and creativity. Each problem is worth seven points, so the perfect score is 42 points.

Formally, like the Olympic Games, the IMO is a competition for *individuals*; participants are ranked according to their score and (multiple) individual medals are awarded. Nevertheless, again as in the Olympic Games, the medals and points obtained by the participants from each country are totaled and the countries are unofficially ranked, providing grounds for comparison between countries.

The two days of heavy problem-solving are followed by a social program for all the participants. Students get to know each other, discuss alternative solutions to the competition problems, and make plans for their future, while the team leaders share their experiences and best practices in creating new problems and preparing their students for the competition.

With its high standards, the IMO prompts the participating countries to constantly improve their educational systems and their methods for selecting and preparing the students. This yielded a great variety of competitions and mathematical enrichment activities around the world which resists any classification. There are "Inclusive" (open for all) competitions which are intended for students of average abilities, while "exclusive" (by invitation only) events target talented students (a prime example of the second type is the IMO and the national olympiad rounds beyond the first). There are "Multiple-choice" competitions where each problem is supplied with several answers, from which the competitor has to find (or guess, as no justification is required) the correct one. In contrast, "classical style" competitions (like the IMO) require the students to present arguments (proofs) in written form. In "correspondence" competitions, such as those organized by *KöMaL* and *Gazeta Matematică*, the students do not necessarily meet each other, while in "presence" competitions (which form the majority of math competitions) the participants are gathered together, which is believed to provide "equal rights" to all students. There are even mixed-style competitions, with a presence-style first stage and correspondence-style subsequent stages. (We will present some newer styles in more detail later.)

Another indication of the importance of the IMO is the fact that other sciences, such as physics, chemistry, and biology, soon followed suit and started international olympiads of their own. Bulgaria organized the first international olympiads in informatics/computer science (1989) and in mathematical linguistics (2003).

3.2. Mathematics competition networks. Like any event with positive social impact, each math competition creates and maintains its network of dedicated people. Numerous math competition journals complement these networks, connecting editorial staff, authors, and readers. Good examples in this direction are Kvant (Russia), Crux Mathematicorum (Canada), Mathematics Magazine and Mathematical Spectrum (UK). The math competition networks range in size from regional to international networks that are associated with large and famous competitions, such as the IMO, Le Kangourou Sans Frontiéres [www.mathkang.org], the Australian Mathematics Competition [www.amt.canberra.edu.au], the International Mathematics Tournament of Towns [www.amt.canberra.edu.au/imtot.html], the Ibero-American Mathematics Olympiad [www.campus-oei.org/oim/], and the Asian-Pacific Mathematics Olympiad [www.cms.math.ca/Competitions/APMO/] – the list is far too short to enumerate all networks that deserve to be mentioned.

The different competition networks are not isolated, as many people naturally belong to more than one network. A different and more formal tie is provided by the *World Federation of National Mathematics Competitions* (WFNMC). The WFNMC was founded in 1984, during the Fifth International Congress of Mathematical Education (ICME5) in Adelaide, Australia. Since then it has a "reserved slot" in the programs of every ICME. Every second year after ICME the WFNMC organizes its own Conference. It has an award, named after Paul Erdös, which is given to people with outstanding contributions to mathematics competitions. The Federation publishes also its journal *Mathematics Competitions* [www.amt.canberra.edu.au/wfnmc.html] which is another powerful tool for networking people engaged with competitions. In 1994 the WFNMC became an Affiliated Study Group of the International Commission on Mathematical Instruction (ICMI), which, in turn, is a commission of the International Mathematical Union [1]. In this way the competitions networks are incorporated into the global mathematical community.

Taken together, these networks form a large global network in the field of mathematics competitions and, more generally, in the classical area known under the (somewhat misleading) name *Elementary Mathematics*. Like in any other area of science, this network operates and lives through its journals, conferences, and workshops, but the periodical regularity of its math competitions adds to its strength and vitality since the people meet more often. In addition, this global network facilitates the dissemination of best practices in curriculum development and in the work with talented youngsters. New problem solving techniques, new classes of problems, and new ideas about organizing competitions spread quickly around the world. We should not forget also that, through this global network, the Elementary Mathematics (which constitutes an important part of our mathematical heritage) is preserved, kept alive and further developed.

4. Why are the competitions needed?

Here is a short and incomplete list of reasons on which we expand later on:

1. higher abilities and talent are identified, motivated and developed;
2. what happens before and after the competition is good for education;
3. talented students are steered to careers in science;
4. competitions raise the reputation of an educational institution.

4.1. Finding higher abilities and talent. The educational systems in most countries target mainly students of average mathematical abilities (who form the majority in schools). Additional care is often provided for lower-ability students, so that they could cover the educational standards. The standard curriculum and syllabus requirements pose no significant challenge however to students with higher abilities. They do not feel the need to work hard and, as a result, their mathematical abilities and talent remain undiscovered and undeveloped.

This is a pity, of course, since these higher-ability youngsters are a very important resource for the development of society, provided they are properly educated, motivated, and supported. Unlike other natural resources, such as mineral deposits, which remain preserved for the future generations, if undiscovered and unused, the talent of a young person is lost forever, if it is not identified, cultivated, and employed properly. Competitions and other enrichment activities are obvious remedies for this shortcoming, as they allow students to exhibit their abilities and talent. Moreover, competitions motivate the participants to work hard while preparing for them and, as a result, further develop their abilities and talent.

4.2. Before and after competitions. Some opponents to competitions complain that there is no apparent direct connection between the competitions and the mathematics as taught in the classroom. This, in our mind, is a rather narrow approach to the issue. Classroom is only one of the many homes of the educational process. One should take into account the integral impact of competitions and competition-related activities on education. What frequently escapes public attention, which often focuses on a rather small group of happy winners, is the fact that the other, “non-winner” participants, also gain a lot. While preparing for the competition, and trying to solve the problems during the competition itself, all participants increase their knowledge significantly. Taking into account that in some competitions hundreds of thousands of students are taking part, the integral impact on the learning of mathematics becomes significant for the overall development of the contemporary society. From this point of view the contribution of the International Competition “European Kangaroo" with more than 3 millions of participants is difficult to overestimate.

We should not neglect also what happens in the corridors of the school (or outside the school) after the competition is over. The students are sharing their experiences (successes, failures, new ideas generated, etc.). This has a tremendous educational effect which however is not always given proper attention. The competitions and mathematics enrichment activities can be viewed as events that provide impetus for subsequent discussions among the students (as well as among their friends, parents, etc.). From the viewpoint of acquiring new mathematical knowledge (facts and techniques) these after competition discussions might be as important as the preparation for and the competition itself. Many of us owe a significant part of our knowledge to just such “corridor mathematics". From this point of view the social program after IMO gains additional importance. All this could (and should) have some practical implications for the ways the competitions and other enrichment activities are planned and organized. One should deliberately incorporate possibilities (the more the better) for “after event" discussions, reflections and interactions. There is an unexhausted potential for introduction and sharing new practices in this area.

Finally, while preparing their students for competitions the teachers gain experience how to teach mathematical topics that are currently not in the curriculum. This may become important at later stages, if some of these topics become a part of the school program.

4.3. Steering talented students to careers in science. The health and longevity of any social sector depends on how many talented young people are attracted to it. The role of math competitions in identifying talented young people and in attracting them to science should be obvious. Indeed, the fact that a significant number of successful participants in math competitions later become famous scientists was recognized rather early. On 17 July 1929, John von Neumann, who was born in Hungary and was influenced by math competitions, wrote in his letter from Berlin to Professor Lipót Fejér in Budapest ([5]):

> Dear highly honored Professor,
>
> I had the opportunity several times to speak to Leo Szilard about the student competitions of the Eötvös Mathematical and Physical Society, also about the fact that the winners of these competitions, so to say, overlap with the set of mathematicians and physicists who later became well-respected world-figures. Taking the general bad reputations of examinations world-wide into account it is to be considered as a great achievement if the selection works with a 50

> percent probability of hitting the talent. Szilard is very interested in whether this procedure can be applied in the German context and this has been the subject of much discussion between us. However, since we would like above all to learn what the reliable statistical details are, we are approaching you with the following request. We would like:
>
> 1. to have a list of names of the winners and runners-ups of the student competitions,
>
> 2. to see marked on the list those who were adopted on a scientific basis and those adopted for other work,
>
> 3. to know your opinion about the extent to which the prizewinner and the talented are the same people and, for example, what proportion of the former would be worthy of financial support from the State in order to make their studies possible.

Very often the future professional realization of a young person is often predetermined by the "first success." The first area where positive results are achieved often becomes the preferred area in which a person invests time and efforts, which in turn brings more success, stronger motivation, and higher professionalism. Math competitions provide such opportunity for early success and thus help attract good young minds to mathematical and scientific careers. In this way competitions contribute to the development and progress of mathematics and other sciences.

4.4. Raising the reputation of an educational institution. The academic reputation of a university depends primarily on the merit of the intellectual achievements of its academic staff. "The higher the reputation of the professors, the higher the reputation of the university" is the essence of this widely accepted belief. What is often overlooked, though, is that the level of the students also has a significant impact on the outcome of the educational process and, in the long run, on the reputation of the institution. While higher-ability students still have the chance of becoming good professionals if trained by ordinary professors, even outstanding professors can fail to produce high-level specialists from mediocre and unmotivated students.

Teachers know well that a few good students in class not only motivate the other students and make them work harder, but also place higher demands on the preparation of the teachers themselves. This two-way challenge influences positively the educational process and improves, directly or indirectly, the reputation of the entire educational institution.

It is no wonder that many universities try hard to attract good students. One of the best ways to achieve this is to organize competitions for secondary school students and to offer incentives, such as stipends or entrance exam waivers, to the winners. Such policies usually yield the expected results, as a special type of relationship develops between organizers and the winners during the preparation for the competition, the competition itself, and the post-competition period, which encourages the winners to consider seriously (sometimes as the first option) enrolling in the university where the competition (and/or the preparation for it) takes place.

In addition to the obvious advantages, enrolling competition winners has a delayed "value-added" effect to the reputation of a university. After graduation, math competition winners, as people with good problem-solving skills, are more likely to get rapid professional recognition, because they are likely to find solutions to difficult and complex real-life problems easier and faster than others. Once their success is noticed and registered by the working environment, the recognition of the problem-solvers' alma mater increases immediately and almost automatically.

As a success story, consider the University of Waterloo, Canada, and the breathtaking rise of its reputation during the seventies and eighties of the last century. Alongside other plausible explanations, such as good management and excellent academic staff, its success can also be attributed to the fact that the University of Waterloo was the host of the Canadian Mathematics Competition [www.cemc.uwaterloo.ca], which attracted a good portion of the best young minds in Canada.

The William Lowell Putnam Competition, widely known as the "Putnam Exam" and administered by the Mathematical Association of America, is the flagship of annual competitions for university students in North America. While enrolled at the University of Waterloo, the former winners in school competitions performed consistently well in the Putnam Exam, securing a prominent presence of Waterloo in the top five teams in North America. This also was contributing to the reputation of the institution. It is no wonder that, within less than 20 years, the University of Waterloo became one of the leading centers for mathematics and computer sciences in the world.

There is another success story related to the University of Waterloo and the Canadian Mathematics Competition, which shows how a new implementation of an inspiring idea at a new place can yield fantastic results.

The Australian mathematician Peter O'Halloran (1931–1994) spent a part of his 1972–73 sabbatical leave from the Canberra College of Advanced Education (now University of Canberra) at the University of Waterloo. There he gained, as Peter Taylor (Executive Director of the Australian Mathematics Trust) recalls ([6]),

> ... the idea of a broadly based mathematics competition for high school students. On his return he often enthused to his colleagues about the potential value of such a competition in Australia. In 1976, while President of the Canberra Mathematical Association, he established a committee to run a mathematics competition in Canberra. This was so successful that the competition became national by 1978 as the Australian Mathematics Competition, sponsored by the Bank of New South Wales (now Westpac Banking Corporation). It is now well known that this competition has grown to over 500,000 entries annually, and is probably the biggest mass-participation event in the country.

The success of Peter O'Halloran was encouraging for others. André Deledicq started in 1991 the Kangaroo Competition in France (the name reveals the Australian influence). The Kangaroo Competition is now truly international (albeit with focus on Europe), enjoying more than 3 million participants each year.

It is an appropriate place here to pay tribute to Peter O'Halloran, who had the vision for the future of mathematics competitions and knew the strategies how to achieve the goals. He understood the role of international collaboration in this field and was the major force behind the inception of WFNMC and its association with ICMI as an Affiliated Study Group.

5. Competitions and science

Before we go any further, we need to consider a natural question:

> Why are math competitions so good in revealing higher mathematical abilities and inclination to doing research?

The simplest and obvious answer seems to be:

> Because both higher abilities and inclination to doing research are *necessary* to be successful in a math competition.

Necessary, but not sufficient. To be successful in a competition, a student often needs not only a good mind, but a very quick one. Most competitions are limited in time to just 3–4 hours, imposing a significant stress on the nervous system of their participants. Not only do students have to solve the problems correctly, they have to do so quickly and in the presence of their direct competitors. Yet, there are many highly creative students, who do not perform well under pressure. Such "slow thinkers" often come up with new and valuable ideas a mere day (or even just five minutes) after the end of the competition, yet receive no reward or incentive.

Traditional competitions disadvantage such students, even though some of them are highly creative and could become good inventors or scientists. Indeed, what matters in science is rarely the speed of solving difficult problems posed by other people. More often, what matters is the ability to formulate questions and pose problems, to generate, evaluate, and reject conjectures, to come up with new and non-standard ideas. All these activities require ample thinking time, access to information resources in libraries or the Internet, communication with peers and experts working on similar problems, none of which are allowed in traditional competitions.

Obviously, other types of competitions are needed to identify, encourage, and develop such special "slower" minds. The competitions should reflect the true nature of research, containing a research-like phase, along with an opportunity to present results to peers – precisely as it is in real science.

As a matter of fact, such competitions, designed to identify students with an inclination to scientific (not only mathematical) research, already exist. Below we present three of them.

5.1. Germany/Switzerland. *Jugend Forscht* (*Youth Quests*) celebrated its 40-th anniversary in 2005. It is a German annual competition for students under the age of 21, who work, alone or in teams, on projects of their own. The projects are presented at special sessions, where the winners are awarded [www.jugend-forscht.de].

Switzerland has a similar competition, which is organized by the *Schweizer Jugend Forscht* (*Swiss Youth Quests*) foundation, established in 1970. The competition, which covers all scientific directions, including social sciences and humanities, has existed since 1967 [www.sjf.ch].

A Google search for the phrase "Jugend Forscht" produced 25 000 hits in 2002; the same search produced half a million hits in 2005! This 20-fold increase speaks for itself, especially since only German language area is included.

5.2. USA. Many such programs exist in the USA. As a matter of fact, Jugend Forscht was originally shaped after the many "Science Fairs" in USA. We mention only one such program here, because it emphasizes mathematics and because it was used as a model for similar programs in other countries. The Virginia-based *Center for Excellence in Education* (CEE) was founded by Admiral H. G. Rickover in 1983. It has the following goals [www.cee.org]:

> The Center for Excellence in Education nurtures careers of excellence and leadership in science and technology for academically talented high school and college students. CEE is as well dedicated to encouraging international understanding among future leaders of the world. CEE's programs challenge students and assist them on a long-term basis to become creators, inventors, scientists and leaders of the 21st century.

The major CEE event, sponsored jointly with the Massachusetts Institute of Technology, is the *Research Science Institute* (RSI) [www.cee.org/rsi/]:

> Each summer approximately 75 high school students gather for six of the most stimulating weeks of their young lives. Selected from the United States and other nations, these students participate in a rigorous academic program which emphasizes advanced theory and research in mathematics, the sciences, and engineering.
>
> Students attend college-level classes taught by distinguished professors. Nationally recognized teachers conduct classes designed to sharpen research skills. In addition, students complete hands-on research with top mentors at corporations, universities, and research organizations.

Only outstanding, carefully selected students are admitted to the program. RSI starts with a series of professional lectures in mathematics, biology, physics, and chemistry. The students are paired with experienced scientists and mentors, who introduce them to interesting research topics and share with them the joy and excitement of exploring new territories. The RSI days are filled with research, evening lectures, ultimate Frisbee, sport events, etc. At the end of the program, the students present their own research, both in written and oral form, and awards are given to the best performers.

The RSI is an international program: almost a third of its students come from other countries. It provides a unique environment for talented students from different parts of the world to meet, live and work together for a relatively long period of time (six weeks seems to be optimal – it is neither too long to become boring nor too short to put unbearable stress). Again, one should not neglect the importance of the networking and friendships fostered by the RSI program for the future development of the participants. The fact that they know each other will make their future collaboration more fruitful. Year after year, the Bulgarian participants in RSI emphasize the social character of the event and the unique atmosphere created during the RSI.

5.3. Bulgaria. Before the 1989 political changes, Bulgaria had a venue for talented young people, very similar to the above-mentioned Jugend Forscht and RSI. It was called *Movement for Technical and Scientific Creativity of the Youth* (abbreviated in Bulgarian to TNTM). Students worked on individual scientific projects and presented their work on special sessions, where winners were awarded. Like almost everything else related with the youth, the TNTM movement was under the umbrella of the Young Communists' League (Komsomol). After the democratization of the Bulgarian society, the Communist League disappeared, along with everything related to it, including TNTM.

A decade later it became absolutely clear that actions were needed to revive those activities at the level of contemporary challenges and requirements. The RSI model was adapted to the conditions in Bulgaria and, as one of the "Year of Mathematics" initiatives, the new *High School Students' Institute of Mathematics and Informatics* (*HSSIMI*) was founded in 2000.

Throughout one academic year, the involved high school students (grade 8–12) work on freely chosen topics (projects) in mathematics and/or informatics (computer science). They work individually or in teams and are supervised by a teacher, a university student, a relative, or just any specialist in the field, willing to help. In fact, some recent HSSIMI projects were successfully supervised by former HSSMI participants, who are now university students.

Warmly accepted by the mathematical community in Bulgaria, the HSSIMI organizes three major events: two competition-like sessions and a *Research Summer School*. The sessions are held at the stand-alone *Students Conference* for High School students in January and at the *School Section* of the Annual Spring Conference of the Union of Bulgarian Mathematicians(UBM) in April. The latter section is actually the most visited section at the Spring Conference of UBM, attended by university professors, researchers, teachers, parents, and school peers.

To participate in the HSSIMI sessions, students submit a written paper with the results of their work. Specialists referee the papers, assess the projects, and suggest improvements. Students present their research at the sessions and winners are awarded. As special award, two of the winners are sent to USA in order to participate in RSI.

The authors of the best projects are invited to a three-week *Research Summer School*. During the first two weeks, eminent specialists from universities, research institutes, and software companies give lectures and practical courses in mathematics and informatics. As in similar programs, the main goal of this preliminary training is to expand the students' knowledge in topics of their interest and to offer new problems for possible projects. During the third week, students hold a High School Students Workshop, where they briefly present their ideas for new projects.

For the short period of its existence, the HSSIMI became a valuable addition to the established (and rather densely populated) system of traditional competitions in Bulgaria. As was planned and expected, the HSSIMI attracted students who were not regulars in the traditional competitions.

Similar initiatives can be found in other countries. There are positive signs of networking between them as well. Good examples in this direction are the Tournament of Towns Summer Conference and the annual International Mathematics Projects Competition (IMPC) in Kazakhstan. Reflecting more closely the nature and spirit of research process, these kinds of activities also attract excellent minds to mathematics and definitely deserve better recognition and support by the professional mathematical communities around the world.

6. What to do next?

In addition to enhancing the traditional math competitions and developing the non-traditional initiatives discussed above, there are other venues for future improvements, such as implementing the current science trends into competitions, targeting other audiences, and supporting and developing the human resources standing behind competitions and other related activities.

6.1. Algorithms in mathematics. The nature of mathematical research has changed significantly since considerable computing power came to the desk of almost every researcher and student. Mathematicians today can conduct complicated numerical experiments, use software for complex algebraic and analytic transformations, find patterns in huge data sets. Like the experiments in other sciences, this could help reject some conjecture or formulate a new one. Thus, research in mathematics became similar to research in the other sciences.

All this is based on mathematical algorithms. Algorithmic thinking is getting higher importance and successfully complements the "axiomatic" approach and thinking in mathematics.

This change should be duly reflected in the creation and selection of competition problems. Perhaps more problems should be offered at various competitions where algorithms and their properties are focused in order to cultivate algorithmic thinking.

Otherwise, we will become witnesses of a "brain-drain" and the best young minds will be driven to competitions in informatics.

6.2. Teamwork. Working in teams is a well-established trend in modern science. For centuries, research in mathematics has been a solitary endeavor. Today, we see more and more teamwork in mathematics and, especially, in its applications. This reveals yet another similarity between modern mathematics and the other sciences (where teamwork has traditionally deeper roots). The ability to work in a team is valuable skill that could and should be cultivated early on.

Mathematics team competitions could contribute a lot in this direction. There are many such competitions around the world; it only makes sense to make them more popular.

6.3. Competitions for university students. Even though they are not the focus of this paper, mathematics competitions for university students, among other virtues, help attract talented young people to academic careers in mathematics.

Some of these university-level competitions are highly respected and have existed for many years. The above-mentioned Putnam Exam is more than 65 years old. The International Mathematics Competition for University Students began in 1994 [www.imc-math.org]. Of course, there are many other such competitions, but their number is still much smaller than the number of competitions for secondary school students, providing plenty of opportunities for new initiatives and international collaboration in this area.

6.4. Teachers and the competitions. In many countries, year after year, some schools consistently "produce" more competition winners than other schools. What is the reason behind this phenomenon? Why are some schools more successful than others?

The reasons may be numerous and fairly different in nature. Very often, however, the prominent success of a particular school can be attributed to the dedicated efforts of a single teacher or a small group of teachers. For these excellent teachers, teaching is a vocation, a mission, and not just means to make both ends meet. Such special teachers are real assets for the school and for the whole country. They possess both the necessary scientific ability and the extraordinary personality needed to identify and motivate for hard work the future winners in competitions.

Such teachers need special care, though. Their higher scientific ability is acquired very slowly, at the expense of great personal efforts. It is no secret that the success of these teachers depends very strongly on their working environment and on the appreciation by their colleagues and administration. Very often however the actual working conditions in the schools do not support the work and the development of these dedicated teachers.

There is a lot that can (and have to) be done in order to improve the situation. For instance, the materials available to the teachers should not include problems and solutions only, but also provide didactic instructions for the teachers how to use these materials in their work with higher ability students and what type of reactions and difficulties to expect on the side of students. For this to happen a special research is needed, conducted with the help of professional math educators.

Many organizations which are involved with competitions are also organizing seminars and workshops for teachers. There is a valuable experience in many countries in the work with such teachers. The positive results and the problems could be discussed and evaluated with the aim of disseminating the good practices. Teachers are the major human resource for the development of competitions and related activities.

Another problem is that often competition-like activities are not "at home" (and therefore not appreciated) both in Mathematics Departments (because "they concern Elementary Mathematics") and in Mathematics Education Departments (because they are "too mathematical and refer to the relatively small group of talented students"). It is time for both communities (research mathematicians and mathematics educators) to understand their joint interest in supporting competitions and competitions – related activities.

7. Summary

Competitions have influenced positively mathematics education and its institutions in different ways for more than a century.

Engaging millions of students and educators, math competitions have a distinguished way to identify, motivate, and develop young talent, steering it to careers in science.

Mathematics competitions have matured and formed an immense and vibrant global network which contributes significantly to the preservation and the maintenance of mathematical heritage.

The flagship IMO not only serves as the "golden standard" for numerous other competitions in mathematics and the sciences (especially with its often-overlooked social program), but it also provides a constant stimulus for improvement of school systems around the world.

Traditional competitions are complemented by more inclusive and less known events that emulate more closely real research and engage even broader student audience.

Nevertheless, stronger consolidation and collaboration of teachers, schools, universities, and educational authorities is needed in order to meet the challenges of the new century.

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