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Invited Lectures
Abstracts

Section 06
Topology

Finiteness of arithmetic Kleinian reflection groups

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2000 Mathematics Subject Classification. Primary 30F40; Secondary 57M.

Key words. Kleinian group, reflection group.

We prove that there are only finitely many arithmetic Kleinian maximal reflection groups.

Non-positive curvature and complexity for finitely presented groups

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2000 Mathematics Subject Classification. Primary 20F65; Secondary 20F67.

Key words. Geometric group theory, finitely presented groups, non-positive curvature, Dehn functions, filling invariants, decision problems.

A universe of finitely presented groups is sketched and explained, leading to a discussion of the fundamental role that manifestations of non-positive curvature play in group theory. The geometry of the word problem and associated filling invariants are discussed. The subgroup structure of direct products of hyperbolic groups is analysed and a process for encoding diverse phenomena into finitely presented subdirect products is explained. Such an encoding is used to solve problems of Grothendieck concerning profinite completions and representations of groups. In each context, explicit groups are crafted to solve problems of a geometric nature.

Link homology and categorification

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2000 Mathematics Subject Classification. 57M25, 57Q45.

Key words. Link homology, quantum link invariants, matrix.

This is a short survey of algebro-combinatorial link homology theories which have the Jones polynomial and other link polynomials as their Euler characteristics.

Curve complexes, surfaces and 3-manifolds

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2000 Mathematics Subject Classification. Primary 57M50; Secondary 30F40, 30F60.

Key words. Kleinian groups, mapping class group, complex of curves.

A survey of the role of the complex of curves in recent work on 3-manifolds and mapping class groups.

$\mathbb{A}^1$-algebraic topology

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2000 Mathematics Subject Classification. 14F05, 19E15, 55P.

Key words. $\mathbb{A}^1$-homotopy theory, Milnor K-theory, Witt groups.

We present some recent results in $\mathbb{A}^1$-*algebraic topology*, which means both in $\mathbb{A}^1$-homotopy theory of schemes and its relationship with algebraic geometry. This refers to the classical relationship between homotopy theory and (differential) topology. We explain several examples of "motivic" versions of classical results: the theory of the Brouwer degree, the classification of $\mathbb{A}^1$-coverings through the $\mathbb{A}^1$-fundamental group, the Hurewicz Theorem and the $\mathbb{A}^1$-homotopy of algebraic spheres, and the $\mathbb{A}^1$-homotopy classification of vector bundles. We also give some applications and perspectives.

Development in symplectic Floer theory

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2000 Mathematics Subject Classification. Primary 53D40; Secondary 53D35.

Key words. Symplectic manifold, Hamiltonian systems, Lagrangian submanifolds, Floer cohomology, A_∞-structure.

In the middle of the 1980s, Floer initiated a new theory, which is now called the Floer theory. Since then the theory has been developed in various ways. In this article we report some recent progress in Floer theory in symplectic geometry. For example, we give an outline of a proof of the flux conjecture, which states that the Hamiltonian diffeomorphism group is C^1-closed in the group of symplectomorphisms for closed symplectic manifolds. We also give a brief survey on the obstruction–deformation theory for Floer theory of Lagrangian submanifolds and explain some of its applications.

Heegaard diagrams and Floer homology

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2000 Mathematics Subject Classification. 53D, 57R.

Key words. Heegaard diagrams, Floer homology, Thurston norm.

We review the construction of Heegaard–Floer homology for closed three-manifolds and also for knots and links in the three-sphere. We also discuss three applications of this invariant to knot theory: studying the Thurston norm of a link complement, the slice genus of a knot, and the unknotting number of a knot. We emphasize the application to the Thurston norm, and illustrate the theory in the case of the Conway link.

The cohomology of automorphism groups of free groups

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2000 Mathematics Subject Classification. Primary 20F65; Secondary, 20F28.

Key words. Automorphism groups of free groups, outer space, group cohomology.

There are intriguing analogies between automorphism groups of finitely generated free groups and mapping class groups of surfaces on the one hand, and arithmetic groups such as $\mathrm{GL}(n,\mathbb{Z})$ on the other. We explore aspects of these analogies, focusing on cohomological properties. Each cohomological feature is studied with the aid of topological and geometric constructions closely related to the groups. These constructions often reveal unexpected connections with other areas of mathematics.