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Invited Lectures
Abstracts

Section 08
Analysis

Quasiconformal geometry of fractals

Mario Bonk

Department of Mathematics, University of Michigan, Ann Arbor, MI 48109, U.S.A.

2000 Mathematics Subject Classification. Primary 30C65; Secondary 20F67.

Key words. Quasiconformal maps, analysis on fractals.

Many questions in analysis and geometry lead to problems of quasiconformal geometry on non-smooth or fractal spaces. For example, there is a close relation of this subject to the problem of characterizing fundamental groups of hyperbolic 3-orbifolds or to Thurston's characterization of rational functions with finite post-critical set.

In recent years, the classical theory of quasiconformal maps between Euclidean spaces has been successfully extended to more general settings and powerful tools have become available. Fractal 2-spheres or Sierpiński carpets are typical spaces for which this deeper understanding of their quasiconformal geometry is particularly relevant and interesting.

Local *Tb* theorems and applications in PDE

Steve Hofmann

Department of Mathematics, University of Missouri, Columbia, MO 65211, U.S.A.

2000 Mathematics Subject Classification. Primary 42B20, 42B25; Secondary 35J25, 35J55, 47F05, 47B44.

Key words. *Tb* theorem, singular integrals, square functions, boundary value problems, Kato problem, layer potentials.

A *Tb* theorem is a boundedness criterion for singular integrals, which allows the L^2 boundedness of a singular integral operator T to be deduced from sufficiently good behavior of T on some suitable non-degenerate test function b. However, in some PDE applications, including, for example, the solution of the Kato problem for square roots of divergence form elliptic operators, it may be easier to test the operator T locally (say on any given dyadic cube Q), on a test function b_Q that depends upon Q, rather than on a single, globally defined b. Or to be more precise, in the applications, it may be easier to find a family of b_Q's for which Tb_Q is locally well behaved, than it is to find a single b for which Tb is nice globally. In this lecture, we shall discuss some versions of local *Tb* theorems, as well as some applications to PDE.

Almost everywhere convergence and divergence of Fourier series

Sergey V. Konyagin

Department of Mechanics and Mathematics, Moscow State University, 119992 Moscow, Russia

2000 Mathematics Subject Classification. Primary 42A20; Secondary 42A24, 42B05, 42C10.

Key words. Trigonometric Fourier series, Walsh system, partial sums, convergence, almost everywhere.

The aim of this expository paper is to demonstrate that there are several challenging problems concerning the behavior of trigonometric Fourier series almost everywhere.

Iterated Segre mappings of real submanifolds in complex space and applications

Linda Preiss Rothschild

Department of Mathematics, University of California at San Diego, La Jolla, CA 92093-0112, U.S.A.

2000 Mathematics Subject Classification. Primary 32H02, 32H35, 32V40; Secondary 14P20.

Key words. Generic submanifolds, holomorphic mappings, Segre mappings, finite determination.

This article is a survey of various applications of the method of iterated Segre mappings obtained by a number of mathematicians, including the author, over the past decade. This method is applied to various problems involving real submanifolds in complex space and their mappings. The article begins with a description of the iterated Segre mappings associated to generic submanifolds. The problems addressed concern transversality of holomorphic mappings, finite jet determination, local stability groups, and algebraicity of holomorphic mappings between real-algebraic manifolds.

Towards conformal invariance of 2D lattice models

Stanislav Smirnov

Université de Genève, Section de Mathématiques, 2–4, rue du Lièvre, Case postale 64, 1211 Genève 4, Switzerland

2000 Mathematics Subject Classification. Primary 82B20; Secondary 60K35, 82B43, 30C35, 81T40.

Key words. Statistical physics, conformal invariance, universality, Ising model, percolation, SLE.

Many 2D lattice models of physical phenomena are conjectured to have conformally invariant scaling limits: percolation, Ising model, self-avoiding polymers, etc. This has led to numerous exact (but non-rigorous) predictions of their scaling exponents and dimensions. We will discuss how to prove the conformal invariance conjectures, especially in relation to Schramm–Loewner evolution.

Aspects of the L^2-Sobolev theory of the $\bar{\partial}$-Neumann problem

Emil J. Straube

Department of Mathematics, Texas A&M University, College Station, TX 77843, U.S.A.

2000 Mathematics Subject Classification. Primary 32W05; Secondary 35N15.

Key words. $\bar{\partial}$-Neumann problem, regularity in Sobolev spaces, compactness, pseudoconvex domains.

The $\bar{\partial}$-Neumann problem is the fundamental boundary value problem in several complex variables. It features an elliptic operator coupled with non-coercive boundary conditions. The problem is globally regular on many, but not all, pseudoconvex domains.

We discuss several recent developments in the L^2-Sobolev theory of the $\bar{\partial}$-Neumann problem that concern compactness and global regularity.

Greedy approximations with regard to bases

Vladimir N. Temlyakov

Department of Mathematics, University of South Carolina, Columbia, SC 29208, U.S.A.

2000 Mathematics Subject Classification. Primary 41A25; Secondary 41A46.

Key words. Nonlinear approximation, greedy algorithm, convergence, best m-term approximation, greedy basis.

This paper is a survey of recent results on greedy approximations with regard to bases. The theory of greedy approximations is a part of nonlinear approximations. The standard problem in this regard is the problem of m-term approximation where one fixes a basis and seeks to approximate a target function by a linear combination of m terms of the basis. When the basis is a wavelet basis or a basis of other waveforms, then this type of approximation is the starting point for compression algorithms. We are interested in the quantitative aspects of this type of approximation. Introducing the concept of best m-term approximation we obtain a lower bound for the accuracy of any method providing m-term approximation. It is known that a problem of simultaneous optimization over many parameters (like in best m-term approximation) is a very difficult problem. We would like to have an algorithm for constructing m-term approximants that adds at each step only one new element from the basis and keeps elements of the basis obtained at the previous steps. The primary object of our discussion is the Thresholding Greedy Algorithm (TGA) with regard to a given basis. The TGA, applied to a function f, picks at the mth step an element with the mth biggest coefficient (in absolute value) of the expansion of f in the series with respect to the basis. We show that this algorithm is very good for a wavelet basis and is not that good for the trigonometric system. We discuss in detail the behavior of the TGA with regard to the trigonometric system. We also discuss one example of an algorithm from a family of very general greedy algorithms that works in the case of a redundant system instead of a basis. It turns out that this general greedy algorithm is very good for the trigonometric system.

Analytic capacity, rectifiability, and the Cauchy integral

Xavier Tolsa

Institució Catalana de Recerca i Estudis Avançats (*ICREA*) *and Departament de Matemàtiques, Universitat Autònoma de Barcelona, Spain*

2000 Mathematics Subject Classification. Primary 30C85; Secondary 42B20, 28A75.

Key words. Analytic capacity, rectifiability, Cauchy transform, Riesz transform, singular integrals.

A compact set $E \subset \mathbb{C}$ is said to be removable for bounded analytic functions if for any open set Ω containing E, every bounded function analytic on $\Omega \setminus E$ has an analytic extension to Ω. Analytic capacity is a notion that, in a sense, measures the size of a set as a non removable singularity. In particular, a compact set is removable if and only if its analytic capacity vanishes. The so-called Painlevé problem consists in characterizing removable sets in geometric terms. Recently many results in connection with this very old and challenging problem have been obtained. Moreover, it has also been proved that analytic capacity is semiadditive. We review these results and other related questions dealing with rectifiability, the Cauchy transform, and the Riesz transforms.