ICM 2006

Invited Lectures
Abstracts

Section 10
Ordinary Differential Equations and Dynamical Systems

On spectral invariants in modern ergodic theory

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2000 Mathematics Subject Classification. Primary 37A, 28D; Secondary 47A05, 47A35, 47D03.

Key words. Ergodic theory, spectral invariants, homogeneous spectrum problem.

This is a short survey of recent developments in one of the oldest areas of ergodic theory, sometimes called the spectral theory of dynamical systems. We mainly discuss the spectral realization problem in the rich class of all invertible measure preserving dynamical systems, a "behavior" of different spectral invariants in natural subclasses of dynamical systems, and a complete solution of Rokhlin's problem on homogeneous spectrum in ergodic theory.

Ergodic Ramsey theory: a dynamical approach to static theorems

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2000 Mathematics Subject Classification. Primary 28D15, 05D10; Secondary 37A45, 37B10.

Key words. Amenable group, ergodic theory, Furstenberg's correspondence principle, nilmanifold, Ramsey theory, recurrence, Stone–Čech compactification.

We discuss classical results of Ramsey theory together with their dynamical counterparts, survey recent developments and formulate some natural open questions and conjectures.

Hyperbolic billiards and statistical physics

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2000 Mathematics Subject Classification. Primary 37D50, Secondary 34C29, 60F.

Key words. Hyperbolic billiards, mixing, limit theorems.

Mathematical theory of billiards is a fascinating subject providing a fertile source of new problems as well as conjecture testing in dynamics, geometry, mathematical physics and spectral theory. This survey is devoted to planar hyperbolic billiards with emphasis on their applications in statistical physics, where they provide many physically interesting and mathematically tractable models.

Braids and differential equations

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2000 Mathematics Subject Classification. Primary 37B30, 35K90; Secondary 34C25, 37L60, 57M25.

Key words. Braids, Conley index, dynamical systems, parabolic PDEs, second order Lagrangian.

Forcing theorems based on topological features of invariant sets have played a fundamental role in dynamics and differential equations. This talk focuses on the recent work of Vandervorst, Van den Berg, and the author using braids to construct a forcing theory for scalar parabolic PDEs, second-order Lagrangian ODEs, and one-dimensional lattice dynamics.

Newton interpolation polynomials, discretization method, and certain prevalent properties in dynamical systems

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2000 Mathematics Subject Classification. 37C05, 37C50, 37D25, 37C29.

Key words. Discretization method, Newton interpolation polynomials, prevalence, pseudotrajectory, growth of number of periodic points, Newhouse phenomenon.

We describe a general method of studying prevalent properties of diffeomorphisms of a compact manifold M, where by *prevalent* we mean true for Lebesgue almost every parameter ε in a generic finite-parameter family $\{f_\varepsilon\}$ of diffeomorphisms on M.

Usually a dynamical property $\mathcal{P}$ can be formulated in terms of properties $\mathcal{P}_n$ of trajectories of finite length n. Let $\mathcal{P}$ be such a dynamical property that can be expressed in terms of only periodic trajectories. *The first idea* of the method is to *discretize* M and split the set of all possible periodic trajectories of length n for the entire family $\{f_\varepsilon\}$ into a *finite number* of approximating periodic pseudotrajectories. Then for each such pseudotrajectory, we estimate the measure of parameters for which it fails $\mathcal{P}_n$. This bounds the total parameter measure for which $\mathcal{P}_n$ fails by a finite sum over the periodic pseudotrajectories of length n. Application of Newton interpolation polynomials to estimate the measure of parameters that fail $\mathcal{P}_n$ for a given periodic pseudotrajectory of length n is *the second idea*.

We outline application of these ideas to two quite different problems:
• Growth of number of periodic points for prevalent diffeomorphisms (Kaloshin–Hunt).
• Palis' conjecture on finititude of number of "localized" sinks for prevalent surface diffeomorphisms (Gorodetski–Kaloshin).

From combinatorics to ergodic theory and back again

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2000 Mathematics Subject Classification. 37A30, 11B25, 37A45.

Key words. Multiple ergodic theorem, multiple recurrence, arithmetic progressions, nilsystems.

Multiple ergodic averages, such as the average of expressions like $f_1(T^n x)$ $f_2(T^{2n}x)\dots f_k(T^{kn}x)$, were first studied in the ergodic theoretic proof of Szemerédi's Theorem on arithmetic progressions. It turns out that all constraints on such averages (in a sense that we describe) have an algebraic character, arising from identities in nilpotent groups. We discuss these averages, several generalizations, and combinatorial implications of the results.

Diagonalizable flows on locally homogeneous spaces and number theory

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2000 Mathematics Subject Classification. 37D40, 37A45, 11J13, 81Q50.

Key words. Invariant measures, locally homogeneous spaces, Littlewood's conjecture, quantum unique ergodicity, distribution of periodic orbits, ideal classes, entropy.

We discuss dynamical properties of actions of diagonalizable groups on locally homogeneous spaces, particularly their invariant measures, and present some number theoretic and spectral applications. Entropy plays a key role in the study of theses invariant measures and in the applications.

All, most, some differentiable dynamical systems

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2000 Mathematics Subject Classification. Primary 37; Secondary 37C, 37D.

Key words. Dynamical system, entropy, Entropy Conjecture, partial hyperbolicity, accessibility, ergodicity, Lyapunov exponent, SRB measure, structural stability, affine diffeomorphism.

In the first part of this paper we study dynamical systems from the point of view of algebraic topology. What features of all dynamical systems are reflected by their actions on the homology of the phase space? In the second part we study recent progress on the conjecture that most partially hyperbolic dynamical systems which preserve a smooth invariant measure are ergodic, and we survey the known examples. Then we speculate on ways these results may be extended to the statistical study of more general dynamical systems. Finally, in the third part, we study two special classes of dynamical systems, the structurally stable and the affine. In the first case we study the relation of structural stability to entropy, and in the second we study stable ergodicity in the homogeneous space context.

Geodesics on flat surfaces

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2000 Mathematics Subject Classification. Primary 57M50, 32G15; Secondary 37D40, 37D50, 30F30.

Key words. Flat surface, Teichmüller geodesic flow, moduli space, asymptotic cycle, Lyapunov exponent, interval exchange transformation, renormalization.

Various problems of geometry, topology and dynamical systems on surfaces as well as some questions concerning one-dimensional dynamical systems lead to the study of closed surfaces endowed with a flat metric with several cone-type singularities. In an important particular case, when the flat metric has trivial holonomy, the corresponding flat surfaces are naturally organized into families which appear to be isomorphic to moduli spaces of holomorphic one-forms.

One can obtain much information about the geometry and dynamics of an individual flat surface by studying both its orbit under the Teichmüller geodesic flow and under the linear group action on the corresponding moduli space. We apply this general principle to the study of generic geodesics and to counting of closed geodesics on a flat surface.

Some recent progress in geometric methods in the instability problem in Hamiltonian mechanics

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2000 Mathematics Subject Classification. Primary 37J40, 37C29, 34C37; Secondary 70H08, 37C50, 34C29,

Key words. Arnol'd diffusion, normally hyperbolic manifolds, whiskered tori, homoclinic intersections.

We discuss some geometric structures that lead to instability in Hamiltonian systems arbitrarily close to integrable. The structures covered in this report are joint work with A. Delshams, T. M. Seara and M. Gidea.

From Brouwer theory to the study of homeomorphisms of surfaces

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2000 Mathematics Subject Classification. Primary 37E30, 37E35, 37J10; Secondary 37E45.

Key words. Brouwer homeomorphism, Hamiltonian homeomorphism, periodic point, foliation on a surface, rotation number.

We will state an equivariant foliated version of the classical Brouwer Plane Translation Theorem and will explain how to apply this result to the study of homeomorphisms of surfaces. In particular we will explain why a diffeomorphism of a closed oriented surface of genus ≥ 1 that is the time-one map of a time dependent Hamiltonian vector field has infinitely many periodic orbits. This gives a positive answer in the case of surfaces to a more general question stated by C. Conley. We will give a survey of some recent results on homeomorphisms and diffeomorphisms of surfaces and will explain the links with the improved version of Brouwer's theorem.