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Short Communications
Abstracts

Section 11
Partial Differential Equations
On fuzzy integral equation of fractional orders

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We present an existence and uniqueness theorem for integral equations of fractional orders involving fuzzy set valued mappings of a real variable whose values are normal, convex, upper semicontinuous and compactly supported fuzzy sets in $\mathbb{R}^n$. The method of successive approximation is the main tool in our analysis.

References

Spectral asymptotics and matrix geometry

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We study second-order elliptic partial differential operators acting on sections of vector bundles over a compact manifold with a non-scalar positive definite leading symbol. Such operators, called non-Laplace type operators, appear, in particular, in gauge field theories, string theory as well as models of non-commutative gravity theories. We show that the leading symbol of such an operator naturally defines a collection of Finsler geometries on the manifold, which can be thought of as a non-commutative deformation of Riemannian geometry when instead of a Riemannian metric there is a matrix valued self-adjoint symmetric two-tensor that plays the role of a “non-commutative” metric. It is well known that there is a small-time asymptotic expansion of the trace of the corresponding heat kernel in half-integer powers of time, with the coefficients being the spectral invariants of the operator. We initiate the development of a systematic approach for the explicit calculation of the coefficients of this asymptotic expansion. We compute explicitly the first two heat trace coefficients for manifolds with boundary and the first three coefficients for manifolds without boundary. We propose a non-commutative deformation of the Einstein-Hilbert action functional as a linear combination of the first three spectral invariants. The critical points of this functional naturally define the “non-commutative” generalization of Einstein equations.

References


Phase transitions in linear dynamical systems with applications

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Mathematical models of many physical and biological systems do not return conservation laws used to derive them. For instance, fragmentation and coagulation models, derived from the principle of conservation of mass may have solutions which are not mass conserving. Such phenomena are called ‘gelation’ (in coagulation) and ‘shattering’ (in fragmentation), and are attributed to a phase transition creating a state which is beyond the resolution of the model. Some processes of this type can be modelled as Markov processes and, if such a phase transition occurs, they are referred to as explosive Markov processes. An analytic theory of such processes, developed within the last several years, covers a wider range of applications, and has provided an exhaustive characterization of cases when such phase transitions occur in terms of the properties of the generator of the underlying dynamical system, see [1]. We present the recent results of this theory together with applications to models arising in biology, kinetic theory and other areas of physical sciences.

References

Linear stability of a self-gravitating compressible fluid with a free boundary

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We consider the initial free–boundary value problem for the self-gravitating compressible three dimensional Euler equations with positive mass density at the boundary, for which we prove the linear stability of static background solutions. Our work can be summarised in two essential steps. First, we transform the free–boundary problem into a fixed-boundary problem in the common way by using the Lagrange formulation of Euler’s equations. We then write the resulting system as a first order system of evolution and constraint equations. Second, we enlarge the system including every first derivative of the fluid velocity as a new variable. This procedure leads to a whole class of systems with different evolution equations. One of these systems admits a symmetric hyperbolic formulation of the evolution equations which might be useful for numerical investigations. Another of these systems allows to decouple certain evolution equations, which can then be solved independently. We prove well posedness for the linearization of these equations near a static background. This is done using known results for the initial fixed-boundary value problem for linear symmetric hyperbolic systems. The treatment of the constraints at the boundary turns out to be the most difficult part of our approach.

References


Some asymptotic properties for convection-diffusion equations

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We discuss some asymptotic properties for solutions of general convection-diffusion equations

\[ u_t + \text{div} f(x,t,u) = \text{div}(A(x,t,u)\nabla u), \quad x \in \mathbb{R}^n, \; t > 0, \]

with initial data \( u(x,0) = u_0(x) \in L^p(\mathbb{R}^n), \; 1 \leq p < \infty \). Under suitable assumptions on \( f \) and \( A \), we show

\[ \|u(\cdot,t)\|_{L^2_p} \leq C(n,p)\|u_0\|_{L^p(t\mu(t))^{-\frac{n}{2p}}}, \quad (1) \]

where \( \mu(t) \) is a positive non-increasing function such that \( \langle A(x,t,\xi)\xi,\xi \rangle \geq \mu(t)\|\xi\|^2, \; \forall \xi \in \mathbb{R}^n \). Moreover, we discuss how the bounds (2) can be used to obtain an \( L^\infty \) estimate

\[ \|u(\cdot,t)\|_{L^\infty} \leq K(n,p)\|u_0\|_{L^p(t\mu(t))^{-\frac{n}{2p}}}. \]

Some interesting applications of these properties are also mentioned. Bounds (2) are similar to the ones established in [1] for the one-dimensional case.

References

Asymptotic behaviour of the porous media equation in domains with holes

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2000 Mathematics Subject Classification. 35B40, 35R35, 35K65

Let $G \subset \mathbb{R}^N$ be a bounded open set with smooth boundary and let $\Omega = \mathbb{R}^N \setminus G$. We study the large-time behaviour of the solution to

$$
\begin{cases}
  u_t = \Delta u^m, & (x,t) \in \Omega \times (0, \infty), \\
  u(x,t) = 0, & (x,t) \in \partial \Omega \times (0, \infty), \\
  u(x,0) = u_0(x), & x \in \Omega,
\end{cases}
$$

where $m > 1$. We assume $u_0$ to be in $L^1(\Omega)$, nonnegative in $\Omega$ and compactly supported in $\overline{\Omega}$. We restrict to $N > 2$, since the situation for $N = 1, 2$ is different.

As a first step we show that $u$ decays as $O(t^{-\alpha})$, while its support expands like $O(t^\beta)$, where $\alpha = N/(N(m - 1) + 2)$ and $\beta = 1/(N(m - 1) + 2)$. We scale $u$ according to these rates, $v_{\text{out}}(y, \tau) = t^\alpha u(yt^\beta, t)$, $\tau = \log t$, and prove that $v_{\text{out}}$ converges as $\tau \to \infty$ to the profile $F_{C_*}$ of a particular Barenblatt solution, $B_{C_*}(x, t) = t^{-\alpha}F_{C_*}(xt^{-\beta})$. The constant $C_*$ is determined from the initial data thanks to an explicit conservation law that takes into account the amount of mass lost through the boundary. This amount is given by the projection of the initial data on a function $\Phi$ which is the normalized harmonic function that measures the capacity of $G$. Convergence is uniform in sets $|y| \geq \delta$, i.e., in a wide exterior region up to the free boundary, what is called in Matched Asymptotics the outer limit.

To complete the study we consider what happens in the region near the holes (so-called inner limit). In this case we only have to amplify the solution, keeping the space variable fixed (i.e., a quasi-stationary situation). We prove that the rescaled function $v_{\text{in}}(x, t) = t^\alpha u(x, t)$ converges to a stationary state, $H_C(x) = C^{-m}H(x)$, where $H = (1 - \Phi)$ is the unique solution of

$$
\Delta H = 0, \quad x \in \Omega, \quad H = 0, \quad x \in \partial \Omega, \quad H \to 1, \quad |x| \to \infty.
$$

The free constant $C$ is adjusted through matching with the Barenblatt function which gives the outer behaviour. It turns out that $C = C_*$. Combining the inner and outer descriptions allows us to write a global uniform approximation for the large-time behaviour of the solution.
The asymptotic behaviour of the solution to (1) was studied formally by King in the radial case [1]. Very recently Gilding and Goncerzewicz [2] have performed a rigorous outer analysis by completely different methods.

References


Homogenization of parabolic problems with varying boundary conditions

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Given $\Omega$ a Lipschitz bounded open set of $\mathbb{R}^N$, $N \geq 2$, and $\Gamma_n$ an arbitrary sequence of subsets of $\partial \Omega$, our purpose in this work is to study the asymptotic behavior of the solution $u_n$ of the following linear parabolic problem

$$
\begin{cases}
\partial_t u_n - \text{div} (A\nabla u_n - G_n) = f_n & \text{in } \Omega \times (0,T) \\
u_n = 0 & \text{on } \Gamma_n \times (0,T), \quad (A\nabla u_n - G_n) \nu = 0 & \text{on } (\partial \Omega \setminus \Gamma_n) \times (0,T) \\
u_n(x,0) = 0 & \text{in } \Omega,
\end{cases}
$$

(1)

with $A \in L^\infty(\Omega \times (0,T))^N \times N$ elliptic and $f_n \in L^2(\Omega \times (0,T))$, $G_n \in L^2(\Omega \times (0,T))^N$ such that $f_n$ converges weakly in $L^2(\Omega \times (0,T))$ to a function $f$ and $G_n$ converges strongly in $L^2(\Omega \times (0,T))^N$ to a function $G$.

Under these conditions, we show the existence of a subsequence of $n$, still denoted by $n$, a nonnegative Borel measure $\mu$ which vanishes on the sets of capacity zero and a positive function $b \in L^\infty(\Omega \times (0,T))$, such that for every sequences $f_n$ and $G_n$ as above, the solution $u_n$ of (1) converges weakly in $L^2(0,T;H^1(\Omega))$ and strongly in $L^2(0,T;H^1_{\text{loc}}(\Omega))$ to the unique solution $u$ of the variational problem

$$
\begin{cases}
u_n = 0 & \text{in } \Omega \\
\int_{\Omega} \partial_t u vd\nu + \int_{\Omega} A\nabla u \nabla v d\nu + \int_{\partial \Omega} bv d\mu = \int_{\Omega} f v d\nu + \int_{\Omega} G v d\nu & \text{in } D'(0,T) \\
u \in H^1(\Omega) \cap L^2(\partial \Omega).
\end{cases}
$$

We observe that assuming smoothness enough, the boundary condition satisfied by $u$ is

$$
(A\nabla u - G) \nu + bu \mu = 0 \text{ on } \partial \Omega.
$$

References


Existence of solutions for a dynamic Signorini’s contact problem

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The purpose of this talk is to present an existence result for the dynamic frictionless contact problem between an elastic body and a rigid foundation. The problem of the wave equation with unilateral boundary conditions has been studied by Lebeau and Schatzman [4] who established the existence and uniqueness of solution for half-space domains. As the authors themselves explain, however, their method cannot be extended to general domains. Smooth bounded domains have been considered by Kim in [2] but his work cannot be applied to elasticity.

In order to model the contact we consider Signorini conditions. So, the contact problem can be posed as follows:

Problem (P) Find \((u, \sigma)\) verifying:

\[
\begin{align*}
\rho \ddot{u} - \text{div}\sigma(u) &= F & \text{in } & \Omega \times (0,T), \\
\sigma n &= g & \text{on } & \Gamma_N \times (0,T), \\
\sigma &= 0 & \text{on } & \Gamma_D \times (0,T), \\
\sigma_t &= 0; \quad \sigma_n \leq 0; \quad u_n \leq 0; \quad \sigma_n u_n = 0 & \text{on } & \Gamma_C \times (0,T), \\
u(x,0) &= u_0; \quad \dot{u}(x,0) = u_1 & \text{in } & \Omega,
\end{align*}
\]

where \(\rho\) is the density of the solid, \(\Omega \subset \mathbb{R}^n\), \(n = 2, 3\) is a bounded domain of class \(C^{1,1}\), \(g \in W^{2,\infty}(0,T; [L^2(\Gamma_N)]^n \cap [H^{-\frac{1}{2}}(\Gamma)]^n)\), \(F \in W^{2,\infty}(0,T; [L^2(\Omega)]^n)\), \(\sigma(u) = \Lambda^{-1}\varepsilon(u)\), \(\Lambda\) being the elasticity tensor assumed to be time independent, symmetric and coercive, \(n\) denotes the unit outward normal vector and \(\text{mes}(\Gamma_D) > 0\). The initial conditions \(u_0\) and \(u_1\) are assumed to belong to \([H^1(\Omega)]^n\) such that \(\text{div}\Lambda^{-1}\varepsilon(u_0) \in [L^2(\Omega)]^n\).

The proof is based on five fundamental steps: a discretization in time, using Newmark’s method, which leads to a discretized problem with unique solution; the construction of functions approximating a solution of the problem; the treatment of the contact condition by means of a Lagrange multiplier whose orthogonality properties allow us to get a priori estimates; the convergence of said functions and, finally, the pass to the limit obtaining a weak solution of the continuous problem.
References


A new method for bounded and blowup solutions of parabolic equations

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We introduce a new method to obtain an a priori estimates, that is, we differentiate the integration of a ratio of two solutions. Several kinds of equations can use this method.

First, we consider the quasilinear parabolic equation

$$
\frac{\partial u}{\partial t} = \nabla \cdot (g(u) \nabla u) + h(u, \nabla u) + f(u) \quad \text{with} \quad u|_{\partial \Omega} = 0, \quad u(x, 0) = \phi(x).
$$

If $f, g$ and $h$ are polynomials with proper degrees and proper coefficients, we will show that the blowup property only depends on the first eigenvalue $\lambda_1$ of $-\Delta$ in $\Omega$ with Dirichlet boundary condition. For the special case,

$$
\frac{\partial u}{\partial t} = \nabla \cdot (u^\alpha \nabla u) + c_1 u^{\alpha-1} |\nabla u|^2 + c_2 u^{\alpha+1}
$$

with $\alpha > 0$ and $c_2 > 0$, our conclusion is: if and only if $c_2(1+\alpha+c_1) > \lambda_1$, then, for any initial value $\phi(x) \in C^{1+\beta}_0(\Omega)$ with $\beta > 0$, the solution blows up in a finite time. The results generalize or complement many previous conclusions in the literature (see [1] and [2]).

Next, we discuss a generalized activator-inhibitor model

$$
u_t = \epsilon \Delta u - \mu u + u^p/v^q \quad \text{and} \quad v_t = D\Delta v - \nu v + u^r/v^s,
$$

with the Neumann boundary conditions. In general, the problem is quite difficult to solve because it has neither a variational structure nor a priori estimates and the comparison principle is failed. By using the new method, we can prove that if $p-1 < r$ and $rq > (p-1)(s+1)$ then for any positive initial values, the positive solutions exist for all time (Jiang [2] also used the similar method to obtain this result). If we add the additional condition $q < s + 1$, then the solutions are $L^\alpha$ bounded for any $\alpha$. We also discuss the uniform bound for the stationary solutions.

Then, we deal with the heat equations

$$
\frac{\partial u_i}{\partial t} = \Delta u_i + f(u_1, \ldots, u_m), \quad u_i(x, 0) = \phi_i(x) \quad \text{for} \quad i = 1, \ldots, m.
$$

Under suitable and natural conditions, the solutions blow up in a finite time. Numerical computations show that the conditions couldn’t be weaken.

References


The mean curvature equation in pseudohermitian geometry

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2000 Mathematics Subject Classification. 35L80, 35J70, 32V20, 53A10, 49Q10

I will make a brief report on the recent study about the mean curvature equation in pseudohermitian geometry ([1], [2]). As a differential equation, this ($p$-)minimal surface equation is degenerate (hyperbolic and elliptic) in dimension 2 while subelliptic in the nonsingular domain for higher dimensions. We analyze the singular set and formulate an extension theorem. This allows us to classify the entire $C^2$-smooth solutions to this equation and to solve a Bernstein-type problem. As a geometric application, we prove the nonexistence of $C^2$-smooth hyperbolic surfaces having bounded $p$-mean curvature, immersed in a pseudohermitian 3-manifold. From the variational formulation of the equation, we study the Dirichlet problem by proving the existence and the uniqueness of the ($p$-)minimizers ([3]).

References


Free vibrations for an asymmetric beam equation

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We consider the semilinear beam equation where the nonlinear term is a functions with some powers:

\[ u_{tt} + u_{xxxx} + bu^+ = f(x, t, u) \quad \text{in} \ (-\pi^2, \pi^2) \times \mathbb{R}, \]

\[ u(\pm \pi^2, t) = u_{xx}(\pm \pi^2, t) = 0, \quad (1) \]

where \( u^+ = \max\{u, 0\} \) and \( f \) is defined by

\[ f(x, t, s) = \begin{cases} |s|^{p-2} s, & \text{amp; } s \geq 0 \\ |s|^{q-2} s, & \text{amp; } s < 0 \end{cases} \quad (2) \]

where \( p, q \geq 2 \) and \( p \neq q \).

McKenna and Walter proved that if \( 3 < b < 15 \) then at least two \( \pi \)-periodic solutions exist, one of which is large in amplitude. The existence of at least three solutions was later proved by Choi, Jung and McKenna, using a variational reduction method. Humphreys proved that there exists an \( \varepsilon > 0 \) such that when \( 15 < b < 15 + \varepsilon \) at least four periodic solutions exist. Choi and Jung suppose that \( 3 < b < 15 \) and \( f \) is generated by eigenfunctions. Since Micheletti and Saccon applied the limit relative category to studying multiple nontrivial solutions for a floating beam.

In this talk, we use a variational approach and look for critical points of a suitable functional \( I \) on a Hilbert space \( H \). Since the functional is strongly indefinite, it is convenient to use the notion of limit relative category.
Standing waves of some coupled nonlinear Schrödinger equations

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In spite of the interest that system of coupled NLS (Nonlinear Schrödinger) equations have in Nonlinear Optics, see e.g. [1], only few rigorous general results have been proved so far. Here, motivated by the recent paper [4], we deal with the system

\[
\begin{aligned}
  -\Delta u + \lambda_1 u &= \mu_1 u^3 + \beta uv^2, \\
  -\Delta v + \lambda_2 v &= \mu_2 v^3 + \beta u^2 v,
\end{aligned}
\]

where \(\lambda_i, \mu_i > 0\), \(i = 1, 2\), \(\beta\) is a real parameter and \(x \in \mathbb{R}^N\), \(N = 2, 3\).

We prove the existence of bound and ground states provided the coupling parameter \(\beta < \Lambda\), respectively, \(\beta > \Lambda'\), where \(0 < \Lambda \leq \Lambda' < \infty\). The main results are the following.

**Theorem 1.** If \(\beta > \Lambda'\) then (1) has a (positive) radial ground state \(\tilde{u}\).

**Theorem 2.** If \(\beta < \Lambda\), then (1) has a radial bound state \(u^*\) such that \(u^* \neq u_j\), \(j = 1, 2\), where \(u_1 = (U_1, 0)\), \(u_2 = (0, U_2)\) and \(U_j\) is the positive radial solution to \(-\Delta U_j + \lambda_j U_j = \mu_j U_j^3\). Furthermore, if \(\beta \in (0, \Lambda)\), then \(u^* > 0\).

The main idea in the proof of Theorems 1, 2 is to show that the Morse index of \(u_1\) and \(u_2\) changes with \(\beta\):

- for \(\beta < \Lambda\) small their index is 1,
- while for \(\beta > \Lambda'\) their index is greater or equal than 2.

This fact, jointly with an appropriate use of the natural constraint method, allows us to prove the existence of bound and ground states.

These results are announced in [2] and proved in [3].

**References**


Regularization method for a free boundary problem of parabolic type modelling the barbotage phenomenon in a molten metal

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We define a piezometric capacity of a saturated zone during a barbotage process and we formulate a Stefan problem; we also derive the theoretical consequences of the directional evolution of the purificator fluid front. The directional barbotage can be expressed as a unilateral problem and allow us to split the unknown function in two components: the directional and residual piezometric capacities, which permit a control of front evolution in saturated zone and formulation of a variational inequality with initial data. We establish by regularization techniques the existence and uniqueness results. In addition, some regularity conditions upon initial data assure us a local / global identification of weak solution with the classical one for the free boundary problem.

References


Standing waves for a generalized Davey-Stewartson system

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A generalized Davey-Stewartson (GDS) system involving three equations was derived in [1], and the study of its qualitative properties was initiated in [2]. Here, we consider the existence of standing waves of GDS system in the purely elliptic case. The relevant equation is

$$\Delta R - \omega R - \chi R^3 - bK(R^2)R = 0,$$

where $\hat{K}(f)(\xi) = \alpha(\xi)\hat{f}(\xi)$ and

$$\alpha(\xi) = \frac{\lambda \xi_1^4 + (1 + m_1 - 2n)\xi_1^2\xi_2^2 + m_2\xi_2^4}{(\lambda \xi_1^2 + m_2\xi_2^2)(\xi_1^2 + m_1\xi_2^2)}.$$

Our main result establishes the existence of solutions for the above equation, hence of the standing waves, under some conditions on the physical parameters $\chi, b, m_1$ and $\omega$. We also establish an alternative sufficient condition for the global existence of solutions to the initial value problem for the GDS system.

References


Optimal $BMO$ estimates near the boundary for solutions of scalar and systems of elliptic problems

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We give the optimal a priori estimates for solutions of scalar regular elliptic boundary value problems, in the general $L^{p,\lambda,s}(\Omega)$ spaces which contain the classical spaces of John and Nirenberg $BMO$, Campanato spaces $L^{2,\lambda}$ and their local versions $bmo$ and $l^{2,\lambda}$ as special cases. This work improves a result of Campanato [1] where he got the estimates for the gradient of solutions of such problems, and it solves a problem raised by Triebel in [4, section 4.3.4]. In the second part we show that this method (used for the scalar case) works for elliptic systems in the sense of Douglis and Nirenberg, and we extend both the scalar case work of the author [3] and the work of Campanato [2] where he obtained the regularity for the gradient of solutions of second order linear strongly elliptic systems.

References

Two remarks on a generalized Davey-Stewartson system

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Many equations can be expressed as a cubic nonlinear Schrödinger (NLS) equation with additional terms, such as the Davey-Stewartson (DS) system [1]. As it is the case for the NLS equation, the solutions of the DS system are invariant under the pseudo-conformal transformation. For the elliptic NLS, this invariance plays a key role in understanding the blow-up profile of solutions, whereas in the hyperbolic-elliptic case of DS system an explicit blow-up profile is obtained via the pseudo-conformal invariance. An analogous system has been derived in [2] to model wave propagation in a generalized elastic medium and has been called Generalized Davey-Stewartson (GDS) system. In [3], for the hyperbolic-elliptic-elliptic and elliptic-elliptic-elliptic cases the GDS system has been expressed as a NLS equation with non-local terms.

We present two results on the GDS system, both following from the pseudo-conformal invariance of its solutions. In the hyperbolic-elliptic-elliptic case, under some conditions on the physical parameters, we establish a blow-up profile. These conditions turn out to be necessary conditions for the existence of a special “radial” solution. In the elliptic-elliptic-elliptic case, under milder conditions, we show the $L^p$-norms of the solutions decay to zero algebraically in time for $2 < p < \infty$.

References


Initial boundary value problems for the KdV equation in fractional-order Sobolev spaces

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Global well-posedness is established for three initial boundary value problems for the Korteweg – de Vries equation

\[ u_t + u_{xxx} + au_x + uu_x = f(t, x) \]

in 1) a right half-strip \( \Pi^+ = (0, T) \times \mathbb{R}_+ \), 2) a left half-strip \( \Pi^- = (0, T) \times \mathbb{R}_- \) and 3) a bounded rectangle \( Q_T = (0, T) \times (0, 1) \) \( T > 0 \) – arbitrary.

Besides an initial condition \( u(0, x) = u_0(x) \) boundary conditions are set, which are different for the considered problems: \( u(t, 0) = u_1(t) \) for the first one, \( u(t, 0) = u_2(t) \), \( u_x(t, 0) = u_3(t) \) for the second one and \( u(t, 0) = u_1(t) \), \( u(t, 1) = u_2(t) \), \( u_x(t, 1) = u_3(t) \) for the third one.

It is assumed, that the initial data \( u_0 \in H^s \), where \( s \geq 0 \) and \( s \neq 3k + 1/2 \), \( k \geq 0 \) – integer, for all three problems and, in addition, \( s \neq 3k + 3/2 \) for the latter two ones. The boundary data \( u_1, u_2 \in H^{(s+1)/3+\varepsilon} \), \( u_3 \in H^{s/3+\varepsilon} \), where \( \varepsilon = 0 \) if \( s \geq 1 \) for the problem in \( \Pi^+_T \), if \( s \geq 2 \) for the problem in \( \Pi^-_T \), if \( s \geq 3 \) for the problem in \( Q_T \), and \( \varepsilon > 0 \) is arbitrary small for the less values of \( s \). Such conditions are natural (or \( \varepsilon \)-close to natural) in the sense, that they originate from properties of solutions to a corresponding initial value problem for a linearized KdV equation \( v_t + v_{xxx} = 0 \).

Solutions of the considered problems are constructed in special functional spaces of Bourgain type. They also preserve \( H^s \) class with respect to \( x \) for every \( t \in [0, T] \) and possess a certain property of extra local smoothing.

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References


Maximal regularity for Kolmogorov operators in $L^2$ spaces with respect to invariant measures

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We are concerned with the differential operator

$$Lu(x) = \frac{1}{2} \sum_{i,j=1}^{d} q_{ij} D_{ij} u(x) + \sum_{i,j=1}^{d} b_{ij} x_j D_i u, \quad x \in \mathbb{R}^d,$$

where $B = (b_{ij})$ and $Q = (q_{ij})$ are real $d \times d$-matrices, $Q$ is symmetric and nonnegative. Therefore $L$ is a possibly degenerate elliptic operator that we assume to be \textit{hypoelliptic}, and that is called Kolmogorov or degenerate Ornstein–Uhlenbeck operator. The hypoellipticity assumption here is the following: the symmetric matrices $Q_t$ defined by $Q_t := \int_0^t e^{sB} Q e^{sB^*} ds$ have nonzero determinant for all $t > 0$. This implies that the Gaussian measures $N_{e^{tB}x,Q_t}$ with covariance operator $Q_t$ and mean $e^{tB}x$ ($t > 0, x \in \mathbb{R}^d$) are all absolutely continuous with respect to the $d$-dimensional Lebesgue measure.

With the aid of such measures the Ornstein–Uhlenbeck semigroup $(T(t))_{t \geq 0}$ is readily defined by $(T(t)f)(x) = \int_{\mathbb{R}^d} f dN_{e^{tB}x,Q_t}.$

Together with hypoellipticity, the other structural assumption is existence of an invariant measure for $L$, i.e., a probability measure $\mu$ such that $\int_{\mathbb{R}^d} Lu d\mu = 0$ for all $u \in C^2_b(\mathbb{R}^d)$. The Ornstein–Uhlenbeck semigroup is then strongly continuous on $L^2(\mathbb{R}^d, \mu)$ and its infinitesimal generator $(L, D(L))$ is a “realization” of the above operator $L$.

Our main theorem states that the domain of $L$ is continuously embedded in the fractional weighted anisotropic Sobolev space $H^{2,2/3, \ldots, 2/(2n-1)}(\mathbb{R}^d, \mu)$. Here the anisotropic splitting of the space comes from the hypoellipticity assumption.

Since our weighted Lebesgue and Sobolev spaces are locally equivalent to the usual Lebesgue and Sobolev spaces, it follows that for each $u \in D(L)$ there exist the derivatives $D_i u$, $D_{ij} u$ for $i, j \in I_0$ ($I_0$ is the first group of variables in the above “anisotropic decomposition”) and they are in $L^2_{loc}(\mathbb{R}^d, dx)$; moreover $u \in H^{2,(2n-1)}_{loc}(\mathbb{R}^d, dx)$. The last exponent $2/(2n-1)$ agrees with the general local regularity results of [2]. Concerning local maximal regularity, we mention also the paper [1] where it was proved that the second order derivatives $D_{ij} u$, $i, j \in I_0$, exist and belong to $L^2_{loc}(\mathbb{R}^d, dx)$. 

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Some global differentiability results for solutions of nonlinear elliptic problems with controlled growths (work in progress)

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Let Ω be a bounded open subset of $\mathbb{R}^n$, $n > 2$. In Ω we deduce the global differentiability result

$$u \in H^2(\Omega, \mathbb{R}^N)$$

for the solution $u \in H^1(\Omega, \mathbb{R}^N)$ of the Dirichlet problem:

$$u - g \in H^1_0(\Omega, \mathbb{R}^N)$$

$$- \sum_i D_i a^i(x, u, Du) = B_0(x, u, Du)$$

with controlled growths and non linearity $q = 2$. In particular we assume that $\forall (x, u, p) \in \Lambda = \Omega \times \mathbb{R}^N \times \mathbb{R}^{nN}$ with $|u| \leq k$

$$\|B^0(x, u, p)\| \leq f_0(x) + c(k)\{|u|^\alpha + \sum_j \|p^j\|\gamma\}$$

where $f_0 \in L^2(\Omega)$, $\alpha \leq \frac{n}{n-2}$, $\gamma \leq \frac{n+2}{n}$.

In the first part of this paper we obtain this result in a particular case ($\gamma = 1$) of this controlled growths, taking into account a local differentiability result proved by S. Campanato, achieving later a differentiability theorem near the boundary and then the global differentiability result using covering procedure. Afterwards we obtain a more general differentiability result ($\gamma \leq \frac{n+2}{n}$) first proving a local differentiability result for a solution $u \in H^1(\Omega, \mathbb{R}^N) \cap C^{0,\lambda}(\Omega, \mathbb{R}^N)$ of the problem (1) making use of interpolation theorem in Besov’s spaces and embedding theorem of Gagliardo-Niremberg type for functions $u \in W^{m,r} \cap C^{s,\lambda}$; proceeding analogously to the first part we obtain the global differentiability result.

References


Behavior of solutions of the first mixed problem for the second order parabolic equations

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Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$ be an arbitrary unbounded domain, $x = (x_1, \ldots, x_n)$ be a point of this space. In the cylindrical domain $D = \Omega \times (t > 0)$ consider the first mixed problem for the second order linear parabolic equation

$$\frac{\partial u}{\partial t} = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} \left( a_{ij}(x,t)u_{x_j} \right) + \sum_{i=1}^{n} b_i(x,t) \frac{\partial u}{\partial x_i} + c(x,t)u$$

(1)

$$u|_{\partial \Omega} = 0$$

(2)

$$u|_{t=0} = \varphi(x)$$

(3)

In the given paper the questions on stabilization of solutions of the first mixed problem for linear divergent parabolic equations with the lowest coefficients are considered. Conditions that connect the coefficients of equation with the geometry of the domain, providing the estimation of solutions are found. In this paper we indicated the class of domains, for which the estimation dependent on the geometry of domain is established. The lower and upper estimations of the solutions, which show an exact order of growth of the solution, are obtained. We proved the Harnack inequality under the definite conditions on the coefficients, connected with the geometry of domain. The Cauchy problem is also considered.

In the given paper the new method for linear divergent equations is practically suggested. These results are extended on nonlinear divergent parabolic equations with the lowest terms.

The questions of stabilization for linear and nonlinear divergent equations were studied by A. K. Gushin [1], F. Kh. Mukminov, I. M. Bikkulov [2], A. F. Tedeev [3], T. S. Gadjiev, Sh. N. Mammadova [4], S.Kamin [5].

References


A priori estimates for two-dimensional parabolic equations

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The motivation of this talk is a series of recent results obtained concerning one-dimensional Cauchy problems with variable domains of operational coefficients, we can mention the works [4, 5]. In this work we study a two-dimensional (in time) Goursat boundary value problem generated by a class of parabolic differential equations with operational coefficients possessing variable domains. We assume that the operational coefficients are submitted to certain conditions in a Hilbert space $H$, then we show that this problem is well posed in Hadamard sense. The proofs are performed by generalization of the well-known method of energy inequalities, first we derive a priori estimates for strong solutions with the use of Yosida operator approximation, and then by using previous results, we show that the range of the operator generated by the posed problem is dense. More precisely, we solve the following problem:

\[
\begin{cases}
\mathcal{L}u = \frac{\partial u}{\partial t_1} + \frac{\partial u}{\partial t_2} + A(t)u = f(t) \\
l_1u(t_1, t_2) = u(t_1, 0) = \varphi(t_1); l_2u(t_1, t_2) = u(0, t_2) = \psi(t_2),
\end{cases}
\]

where $t = (t_1, t_2) \in D$ which is a bounded rectangle in $\mathbb{R}^2$, $f$, $\varphi$ and $\psi$ are given.

In the case where the operational coefficients have constant domains, various important results were proved under different assumptions, see [1, 2]. In all these works, the proofs are obtained via a priori estimates, which follow from the energy inequalities method. Our results extend some ones obtained in [3, 4]. A similar technique as in [3-5] will be applied in this study; however, the results aim at different type of equations and arise from distinct hypotheses.

References


Elliptic problems with nonlocal boundary-value conditions

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We discuss various settings of elliptic problems with nonlocal boundary-value conditions of the following type:

\[
P(x, D_x)u = f_0(x), \quad x \in Q, \]

\[
B_j(x, D_x)u + N_j u = f_j(x), \quad x \in \partial Q, \quad j = 1, \ldots, m,
\]

where \(Q \subset \mathbb{R}^n\) is a bounded domain, \(\{P(x, D_x), B_1(x, D_x), \ldots, B_m(x, D_x)\}\) corresponds to “local” elliptic boundary-value problem (with \(P(x, D_x)\) being an elliptic operator of order \(2m\)), and \(N_j\) are nonlocal operators. The boundary of \(Q\) may contain angular points or edges. The nonlocal operators \(N_j\) need not be small or compact perturbations.

T. Carleman (1932), M. Vishik (1952), F. Browder (1964), A. V. Bitsadze and A. A. Samarskii (1969) were the first to study nonlocal elliptic problems. Significant progress in the general theory of nonlocal problems has been made by A. L. Skubachevskii (since the 1980s).

In this communication, a special attention is paid to the most complicated situation where the support of nonlocal terms can intersect the boundary \(\partial G\). The main difficulties here are due to the fact that generalized solutions of such a problem can have power-low singularities near some points of the boundary \(\partial G\) even if the boundary and the right-hand side \((f_0, f_1, \ldots, f_m)\) are infinitely smooth [1, 2].

We formulate theorems on the (Fredholm) solvability of nonlocal problems and give necessary and sufficient conditions under which the regularity of generalized solutions does not fail [2].

Applications to the plasma theory, control theory, biophysics, and theory of multidimensional diffusion processes are mentioned.

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References

On the vibrations of a thin plate with a concentrated mass

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We look at the vibrations of an elastic, homogeneous and isotropic plate that contains a small region whose size depends on a small parameter $\varepsilon$. The density is of order $O(\varepsilon^{-m})$ in the small region, the so-called concentrated mass, and it is of order $O(1)$ outside; $m$ is a positive parameter. We assume that the thickness plate $h_0$ also depends on $\varepsilon$; indeed, we perform the study in the case where $h = \varepsilon^r$ with $h^2 = h_0^2/12$ and $r \geq 1$.

We consider the associated spectral problem in the framework of the Reissner–Mindlin plate model, which takes into account the thickness plate. We address the asymptotic behavior of the eigenvalues $\zeta^\varepsilon$ and the eigenfunctions $u^\varepsilon$ of this spectral problem when the parameter $\varepsilon$ tends to zero. Depending on the values of $m$ and $r$, there appear different limit behaviors for the eigenelements that we describe. We use methods of spectral perturbation theory along with techniques of matched asymptotic expansions, which allow us to provide additional information on the structure of the eigenfunctions associated with the low frequencies (see [1]–[3]).

References

Exact solutions for the generalized shallow water wave equation by the general projective riccati equation method

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Conte [1] presented a general ansatz to seek more new solitary wave solutions of some NLPDEs. More recently, Yan [4] developed Contes method and presented the general projective Riccati equations method. In this paper we will use this method to construct more exact solutions (soliton and periodic) for the generalized shallow water wave and the combined cosh-sinh Gordon equation. We present a programm in mathematica and applied the method to search solutions of NLPDEs.

Key words: Nonlinear differential equation; Travelling Wave Solution; Mathematica; Projective Riccati Equation Method.

References


On the stability of the minimal solution to quasilinear degenerate elliptic equation

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We shall investigate the stability of the minimal solutions of quasilinear elliptic equation with Dirichlet boundary condition given by

\[
\begin{aligned}
L_p(u) &= \lambda f(u), \quad \text{in } \Omega, \\
u &= 0, \quad \text{on } \partial \Omega,
\end{aligned}
\]

where \( \lambda \) is a nonnegative parameter, \( \Omega \) is a bounded domain of \( \mathbb{R}^N \) and \( L_p(\cdot)(p > 1) \) is the \( p \)-Laplace operator defined by \( L_p(\cdot) = -\text{div}(|\nabla \cdot|^{p-2} \nabla \cdot) \).

We assume that \( f(t) \) is increasing on \([0, \infty)\) and strictly convex with \( f(0) > 0 \). Here the minimal solution \( u_\lambda \) is defined as the smallest solution among all possible classical solutions. Then we shall establish the stability of the minimal solutions \( u_\lambda \) for any \( \lambda > 0 \), that is to say, the nonnegativity of the first eigenvalue the linearized operator \( L'_p(u_\lambda)(\cdot) - \lambda f'(u_\lambda) \) on \( L^2(\Omega) \) will be shown. In the lecture we shall give a similar result on the Bi-\( p \)-Laplace operator \( M_p(u) = \Delta(|\Delta u|^{p-2}\Delta u) \) as well.

**Theorem (P-Harmonis case)** Let \( u_\lambda \) be the minimal solution for \( \lambda \in (0, +\infty) \). Then the first eigenvalue of \( L'_p(u_\lambda) - \lambda f'(u_\lambda) \) is nonnegative. In other words, we have the Hardy type inequality:

\[
\int_\Omega |\nabla u_\lambda|^{p-2}\left(|\nabla \varphi|^2 + (p-2)\frac{(\nabla u_\lambda, \nabla \varphi)^2}{|\nabla u_\lambda|^2}\right) dx \geq \lambda \int_\Omega f'(u_\lambda)\varphi^2 dx,
\]

(1)

for any \( \varphi \in V_{\lambda,p}(\Omega) \). Here \( V_{\lambda,p}(\Omega) \) is defined by

\[
V_{\lambda,p}(\Omega) = \{ \varphi \in M(\Omega) : ||\varphi||_{V_{\lambda,p}} < +\infty, \varphi = 0 \text{ on } \partial \Omega \},
\]

(2)

\[
||\varphi||_{V_{\lambda,p}}^2 = \int_\Omega |\nabla u_\lambda(x)|^{p-2}|\nabla \varphi|^2 dx
\]

(3)

and by \( M(\Omega) \) we denote the set of all measurable functions on \( \Omega \).

**References**

Limit relative category applied to the critical points result for the nonlinear Hamiltonian system

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Let $H(z(t))$ be a $C^1$ function defined on $\mathbb{R}^{2n}$. Let $z = (p, q)$, $p = (z_1, \cdots, z_n)$, $q = (z_{n+1}, \cdots, z_{2n})$. In this paper we investigate the multiplicity of $2\pi$-periodic solutions of the following Hamiltonian system

$$
\dot{p} = -H_q(p, q), \quad \dot{q} = H_p(p, q).
$$

The system can be written in a compact version

$$
\dot{z} = J(H_z(z)), 
$$

where $z : \mathbb{R} \to \mathbb{R}^{2n}$, $\dot{z} = \frac{dz}{dt}$, $J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$, $I$ is the identity matrix on $\mathbb{R}^n$, $H : \mathbb{R}^{2n} \to \mathbb{R}$, and $H_z$ is the gradient of $H$. Let $E = W^{1,2}([0, 2\pi], \mathbb{R}^{2n})$. We look for the weak solutions $z = (p, q) \in E$ of (1.1); that is, $z = (p, q)$ satisfies

$$
\int_0^{2\pi} \left[(\dot{p} + H_q(z)) \cdot \psi - (\dot{q} - H_p(z)) \cdot \phi\right] dt = 0
$$

for all $w = (\phi, \psi) \in E$. Let $a \cdot b$ and $| \cdot |$ denote the usual inner product and norm on $\mathbb{R}^{2n}$. Assume that $H$ satisfies the following conditions:

$(H1)$ $H \in C^1(\mathbb{R}^{2n}, \mathbb{R})$, $H(z) = o(|z|^2)$ as $|z| \to 0$.

$(H2)$ There exist $1 < p_1 \leq p_2 < 2p_1 + 1$, $\alpha_i > 0$, $\beta_i \geq 0$ for $i = 1, 2$ such that

$$
\alpha_1 |z|^{p_1+1} - \beta_1 \leq H(z) \leq \alpha_2 |z|^{p_2+1} + \beta_2 \quad \text{for every } z \in \mathbb{R}^{2n}.
$$

$(H3)$ There exist $0 < \frac{p_2}{2} < q_1 \leq q_2 < 2$ and $\alpha_i$, $\tau_i > 0$, $\beta_i \geq 0$ for $i = 1, 2$ such that

$$
\alpha_1 e^{\tau_1 |z|^{q_1}} - \beta_1 \leq H(z) \leq \alpha_2 e^{\tau_2 |z|^{q_2}} + \beta_2 \quad \text{for every } z \in \mathbb{R}^{2n}.
$$

Our main results are as follows.
Theorem 1.1. Assume that $H$ satisfies the conditions $(H_1), (H_2)$. Then the Hamiltonian system (1.1) has at least two nontrivial $2\pi$-periodic solutions.

Theorem 1.2. Assume that $H$ satisfies the conditions $(H_1), (H_3)$. Then the Hamiltonian system (1.1) has at least two nontrivial $2\pi$-periodic solutions.
Representation Green function of the Dirichlet problems for the bi-harmonic equation

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In the theory of elasticity explicit representations of solutions of the Dirichlet problems for bi-harmonic of the equation plays an important role. Let us consider the following problem.

Find in an area $\Omega_\delta = \{x : \|x - x_0\| < \delta\} \subset \mathbb{R}^n, n = 2m + 1$ a function $u(x)$, satisfying to the following equation and boundary conditions:

$$\Delta^2 u(x) = f(x), \quad u|_{\partial \Omega_\delta} = 0, \quad \frac{\partial}{\partial n_x} u|_{\partial \Omega_\delta} = 0,$$

where $\partial \Omega_\delta = S_\delta = \{x : \|x\| = \delta\}$.

H. Begehr et others [1] for the first time explicitly solved a certain Dirichlet problem for the inhomogenous polyharmonic equation in the unit disc of the complex plane. But the problem of representation of Green function in explicit form was remained unsolved.

The fundamental solution of the equation has a form [2] and be using the property of symmetry [3]:

$$|x - \frac{y}{|y|^2} \delta^2|^{4-n} \frac{y}{\delta}^{4-n} = |y - \frac{x}{|x|^2} \delta^2|^{4-n} \frac{x}{\delta}^{4-n}$$

we proved the following theorem:

**Theorem.** The Green function of the Dirichlet problem can be represented as:

$$G_{n,2}(x, y) = \frac{1}{(4-n)4(2-n)} \cdot \frac{1}{(2\pi)^n} \cdot \frac{1}{|x - y|^{4-n} - |x - \frac{y}{|y|^2} \delta^2|^{4-n} - \frac{y}{|y|^2} \delta^2|^{4-n}}$$

$$+ \frac{1}{2 \cdot 4 \cdot (2-n)} \cdot \frac{1}{(2\pi)^n} \cdot \delta^2 \left[1 - \frac{|y|^2}{|\delta|^2}\right] \left[1 - \frac{|\delta|^2}{|y|^2}\right] |x - \frac{y}{|y|^2} \delta^2 - \frac{y}{|y|^2} \delta^2|^{2-n} \frac{|y|^2}{|\delta|^2}^{2-n}.$$
References


Some existence result to quasilinear elliptic equations

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2000 Mathematics Subject Classification. 35D05

Consider the quasilinear elliptic equation

$$-\sum_{i,j=1}^{N} a_{ij}(x,u) \frac{\partial^2 u}{\partial x_i \partial x_j} + f(x,u,\nabla u) = 0$$

on a bounded smooth domain $\Omega$ in $\mathbb{R}^N$ with $|f(x,r,\xi)| \leq h(|r|)(1 + |\xi|^\theta), 0 \leq \theta < 2$. We note that if there exist a super-solution $\psi$ and a sub-solution $\phi$ with $\psi, \phi \in W^{1,\infty}(\Omega)$ and $\psi \geq \phi$ a.e., then the existence of solutions is irrelevant to $a_{ij}(x,r)$ for $r$ beyond $I_0 = [-\|\psi\|_\infty, \|\phi\|_\infty]$. It is shown that if the oscillation $a_{ij}(x,r)$ with respect to $r$ are sufficiently small for $r \in I_0$ then there exists a solution $u \in W^{2,p}(\Omega) \cap W^{1,p}_0(\Omega), 1 \leq p < \infty$.

References


Kawahara equation in bounded domains

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Initial boundary value problems in $Q = (0, T) \times (0, 1)$ for the Kawahara equation

$$u_t + uu_x + aD_x^3u - bD_x^5u = 0,$$

where $a, b$ are constant coefficients, are considered. When $b = 0, a = 1$ we have the KdV equation. The Kawahara equation describes the dynamics of long waves in a viscous fluid [1] and was considered in $x \in R$ by various authors. On the other hand, mixed problems in bounded domains were not studied. However, they have physical sense and are interesting as mathematical objects since boundary conditions for this equation of odd order are not symmetric and a type of a corresponding mixed problem depends on a sign of the coefficient $b$.

First we consider in $(0, 1)$ boundary value problems for the stationary Kawahara equation

$$cu_t + uu_x + aD_x^3u - bD_x^5u = f(x),$$

where $c$ is a positive constant, $b$ may be positive or negative, which defines a corresponding boundary problem, and $ab < 0$. We prove the existence, uniqueness and continuous dependence of regular solutions on $f(x)$. Then using discretization with respect to $t$ and discrete Gronwall lemma, we prove the existence of local regular solutions to the mixed problem for (1). Global regular estimates show that these local solutions may be prolonged for any finite $T > 0$. Finally, the exponential decay of $L^2$-norms of solutions as $t$ tends to infinity is proved.

References

The resonant nonlinear Schrödinger equation and the reaction-diffusion system and the applications

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2000 Mathematics Subject Classification. 35Q51, 35Q55

I will make a brief report on the resonant nonlinear Schrödinger equation and the Reaction-Diffusion system and the application[1], [2]. A novel integrable version of the NLS equation namely,

$$i \frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + \Lambda \frac{1}{4} |\psi|^2 \psi = s \frac{1}{|\psi|} \frac{\partial^2 |\psi|}{\partial x^2} \psi.$$  (1)

has been termed the resonant nonlinear Schrödinger equation (RNLS)[1]. It can be regarded as a third version of the NLS, intermediate between the defocusing and focusing cases. The additional nonlinear term in the NLS can be viewed as due to an additional electrostriction pressure or diffraction term [4]. Even though the RNLS is integrable for arbitrary values of the coefficient $s$, the critical value $s = 1$ separates two distinct regions of behaviour. Thus, for $s < 1$ the model is reducible to the conventional NLS, (focusing for $\Lambda > 0$ and defocusing for $\Lambda < 0$). However, for $s > 1$ it is not reducible to the usual NLS, but rather to a Reaction-Diffusion system. In this case, the model exhibits novel solitonic phenomena [1].

The RNLS can be interpreted as an NLS-type equation with an additional ‘quantum potential’ $U_Q = |\psi|_{xx}/|\psi|$. It is noted that the RNLS equation, like the conventional NLS equation, may also be derived in the context of capillarity models [2]. The RNLS equation is also related to an equation in plasma physics. It describes the propagation of one-dimensional long magnetoacoustic waves in a cold collisionless plasma subject to a transverse magnetic field[3]. A Hirota bilinear representation of the Reaction-Diffusion system with non-zero boundary condition is given. Here one dissipaton and two-dissipaton exact solutions are obtained by Hirota bilinear method. Some plots of solutions will be also presented here. The results presented are joint works with O. K. Pashaev.

References


Quasineutral limits of the Vlasov-Maxwell-Fokker-Planck system

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In the talk, we shall discuss the quasineutral limits of the Vlasov-Maxwell-Fokker-Planck (VMFP) system. It provides a statistic description of plasma in the terms of charged particle density \( f_\epsilon(t,x,\xi) \) depending on the time \( t \geq 0 \), the position \( x = (x_1, x_2, x_3) \in [0, 1]^3 \equiv \mathbb{T}^3 \), and the velocity \( \xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3 \). The (rescaled) VMFP system takes the form

\[
\partial_t f_\epsilon + \xi \cdot \nabla_x f_\epsilon + (E_\epsilon + \alpha \xi \times B_\epsilon) \cdot \nabla_\xi f_\epsilon = \text{div}_\xi (\beta \xi f_\epsilon + \sigma \nabla_\xi f_\epsilon),
\]

\[
\epsilon^2 \alpha \partial_t E_\epsilon - \text{curl}_x B_\epsilon = -\alpha \int_{\mathbb{R}^3} \xi f_\epsilon \, d\xi,
\]

\[
\alpha \partial_t B_\epsilon + \text{curl}_x E_\epsilon = 0,
\]

where \( \epsilon, \alpha, \beta, \sigma \) are positive parameters.

Under an appropriate on initial data, we show that, as \( \epsilon \to 0 \), the solution of the above VMFP system converges to the so-called electron magnetohydrodynamics equations

\[
\partial_t v + \text{div}_x (v \otimes v) - \epsilon - \alpha v \times b + \beta v = 0,
\]

\[
\alpha \partial_t b + \text{curl}_x e = 0, \quad \text{div}_x v = 0
\]

\[
\alpha v - \text{curl}_x b = 0, \quad \text{div}_x b = 0.
\]

In fact, we obtain that the current \( J_\epsilon = \rho_\epsilon \frac{\partial}{\partial t} \epsilon \to 0 \) of the above VMFP system converges weakly to \( v \) in \( L^\infty([0,T], L^1(\mathbb{T}^3)) \), the scaled electric field and the magnetic field converge strongly

\[
\epsilon E_\epsilon \to 0 \quad \text{and} \quad B_\epsilon \to b \quad \text{in} \quad L^\infty([0,T], L^2(\mathbb{T}^3))
\]

respectively as \( \epsilon, \sigma \to 0 \).

The details can be found in [3].

References

Global Entropy Solutions for a Nonstrictly Hyperbolic System

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2000 Mathematics Subject Classification. 20K

An important class of the equations arising in applications are the nonlinear system of conservation laws. The basic question in this area is the existence of solutions to these equations. This helps to answer the question if the model of the natural phenomena at hand has been done correctly, if the problem is well posed.

In the past years, we studied the existence of weak solutions for some nonlinear hyperbolic equations arisen in the compressible fluid and gas mechanics. In this talk, we shall introduce an existence theorem for global entropy solutions to the nonstrictly hyperbolic system

\[
\begin{align*}
\rho_t + (\rho u)_x &= 0 \\
\frac{\theta}{2} u_x + (\frac{\theta}{2} u^2 + P(\rho))_x &= 0,
\end{align*}
\]

with bounded measurable initial data

\[
(\rho(x, 0), u(x, 0)) = (\rho_0(x), u_0(x)), \quad \rho_0(x) \geq 0,
\]

where the nonlinear function \( P(\rho) = \theta \rho^{\gamma-1}, \theta = \frac{\gamma-1}{2} \) and \( \gamma > 3 \) is a constant.

The method we use here is the theory of compensated compactness coupled with some basic ideas of the kinetic formulation by Lions, Perthame, Souganidis and Tadmor[1, 2]

References


Unified approach to solve inverse problems for partial differential equations

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The direct variational method is used successfully to solve a variety of inverse problems. The types of the governing differential equations considered cover a wide spectrum, including elliptic, parabolic, systems of equations, etcetera. In addition, the auxiliary conditions range from boundary conditions, where the boundary is fixed, to cases where the boundary is free or moving. Incorporating multi-objective optimization in connection with the variational approach, inverse problems with auxiliary conditions specified at randomly chosen points of the region can be tackled. The essence of the method is to: i) Find a functional whose critical point is a solution for the differential equation. ii) Choose the feasible functions for the variational problem to be functions satisfying the given boundary, initial, and auxiliary conditions. iii) Incorporate the unknown boundary (in case of free or moving boundary condition) as one of the unknown functions in the variational problem.

References

Fourth order elliptic equation with critical exponent on complete manifolds

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Let \((M, g)\) be an \(n\)-dimensional complete noncompact Riemannian manifolds with \(n \geq 5\). We use the variational method to investigate existence of solutions \(u \in C^{4, \alpha}_{loc}(M), \alpha \in (0, 1)\) to the fourth order nonlinear elliptic equation

\[ \Delta_g^2 u + \text{div} \left( a(x) \nabla_g u \right) + b(x) = f(x) |u|^{N-2} u \]  

where \(\Delta_g u = -\text{div} (\nabla_g u)\) is the Laplace-Beltrami operator, \(a, b, f\) are smooth functions on \(M\) and \(N = \frac{2n}{n-2}\) is the critical Sobolev exponent. This equation has geometrical roots, it arises from the study of the so-called \(Q\)-curvature which can be interpreted by means of the equation

\[ P_g u = \frac{n-4}{2} Qu^{\frac{n+4}{n-4}} \]

where

\[ P_g = \Delta_g^2 u + \text{div}(a_n S_g + b_n Ric)du + \frac{n-4}{2} Q \]

is the well known Panietz-Branson operator, and

\[ Q = c_n |Ric|^2 + d_n S_g^2 \frac{1}{2(n-2)} \Delta_g S_g. \]

\(S_g, Ric\) denote respectively the scalar and Ricci curvature, and \(a_n, b_n, d_n\) are dimensional constants.

Equation(1) is investigated in the case of compact manifold by D.Caraffa [2] firstly, for \(f\) constant and secondly, for \(f\) a positive function. The author used the variational approach introduced by Yamabe when he attempted to solve his equation.

On complete noncompact Riemannian manifold, conditions on the geometry of the manifold together with conditions on the decay of the function \(f\) at infinity must be added.

We present the main result as follows:
Theorem 1. Let $M$ be a complete noncompact riemannian $n-$manifold ($n > 6$) with bounded geometry, in the sens that the Ricci curvature is bounded from below and the injectivity radius is positive. Let $a, b,$ and $f$ be smooth real valued functions on $M.$ Suppose that the operator $I(u) = \int_M |\Delta_g u|^2 \, dv_g - \int_M a(x)|\nabla_g u|^2 \, dv_g + \int_M b(x)u^2$ is coercive. Under the following assumptions:

1. The functions $a, b, f$ are bounded, $f$ is nonnegative and
   \[ \int_M f(x) \, dv_g < \infty, \int_M a(x) \, dv_g < \infty, \int_M b(x) \, dv_g < \infty, \]

2. There exists a positive constant $C$ such that
   \[ |\Delta_g f| < Cf, \text{ and } \int_M f^{n+1} \, dv_g < \infty, \]

3. For any $\varepsilon > 0$ there exists a compact set $K$ such that $f \leq \varepsilon$ on $M - K,$ and

4. At a point $x_0$ where $f$ is maximal we have
   \[ scal(x_0) + \frac{2(n-1)}{n^2 - 2n - 4} a(x_o) - \frac{n-4}{4(n^2 - 2n - 4)} \Delta_g f(x_0) > 0, \]

There exists a solution $u \in H^2_2(M)$ of (1). Moreover, $u \in C^{1,\beta}_{loc}(M),$ for some $\beta \in (0, 1).$

References


Smooth null-solutions to PDEs with several Fuchsian variables

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N. S. Madi([4]) extended the notion of Fuchsian partial differential equations defined by M. S. Baouendi and C. Goulaouic([1]) to those with several Fuchsian variables. For such an operator $P = P(x,y; D_x, D_y)$ with $(x,y) = (x_1,\ldots,x_p,y_1,\ldots,y_q)$ where $x$ are Fuchsian variables, a distribution $u$ near $(x,y) = (0,0)$ is called a null-solution, if $Pu = 0$ and $(0,0) \in \text{supp } u \subset \{(x,y) | x_j \geq 0(\forall j)\}$. The author and his collaborators([2]) showed the existence of distribution null-solutions under some assumptions, which is also an extension of the result for the case with one Fuchsian variable([3],[5]). There is, however, a big difference between the case with one Fuchsian variable where there are no $C^\infty$ null-solutions, and the case with more Fuchsian variables which include some cases where there can be a $C^\infty$ null-solution. This talk gives some examples and results to make this difference clearer.

One of the main results is about the typical case, where there are only two variables which are both Fuchsian. Let $P = \sum_{i,j \leq m} a_{i,j}(x_1,x_2)D_{x_1}^i D_{x_2}^j$ be a partial differential operator with respect to $x = (x_1,x_2)$ and assume that $P$ is Fuchsian with weight 0 with respect to $(x_1,x_2)$ in the sense of Madi, i.e. $a_{i,j}(x_1,x_2) = x_1^i x_2^j \tilde{a}_{i,j}(x_1,x_2)$ where $\tilde{a}_{i,j}$ are real-analytic near $(0,0)$. We assume that the principal symbol has the form $p_m(x;\xi) = \prod_{l=1}^m (x_1 \lambda_l - \lambda_l(x)x_2 \xi_2)$ where $\lambda_l$ are distinct real-valued real-analytic functions. Then, we have the following.

1. If there exists $l$ such that $\lambda_l(0,0) > 0$, then there exists a $C^\infty$ null-solution $u$.

2. If $\lambda_l(0,0) < 0$ for every $l$, then there exists no $C^\infty$ null-solutions, but there exists a distribution null-solution.

References


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Quasilinear equations with quadratic growth in $\nabla w$ and large solutions for semilinear equations

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In this work we prove the existence of positive solution for the problem

$$-\Delta w + h(w)|\nabla w|^2 = k(x) \text{ en } \Omega$$

$$w \in H_0^1(\Omega)$$

where $\Omega \subset \mathbb{R}^N (N \geq 2)$ is a bounded and smooth domain, $k \in L^\infty(\Omega)$, $k(x) > 0$ and $h : (0, +\infty) \rightarrow [0, +\infty)$ is a function that satisfies

$$\lim_{s \rightarrow 0^+} sh(s) < +\infty.$$

We improve results in [2] to handle singular functions $h$ at $s = 0$.

We will also present an application to the existence of solutions to the boundary value problem having blow-up at the boundary. Specifically, we solve the b.v.p.

$$\Delta u = k(x)f(u), \text{ en } \Omega$$

$$\lim_{\text{dist}(x,\partial\Omega) \rightarrow 0} u(x) = +\infty.$$  

Contrary to the previous works [1, 3, 4] we don’t assume $f$ to be nondecreasing.

References


Mathematical modelling of a nuclear waste disposal site via homogenization

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The goal of our work is to give an accurate model describing the global behavior of an underground waste repository array, made of a high number of units containing waste, once the units start to leak. The purpose of such a global model is to be used for the full field three dimensional simulations necessary for safety assessments. The disposal site can be described as a repository array made of high number of units inside a low permeability layer, called host layer, like for example clay, included between layers with higher permeability (e.g. limestone). There is a flow crossing the repository array which is produced by the pressure drop, or hydraulic head, in the region. The pollutant is then transported both by the convection produced by the water flowing slowly (creeping flow) through the rocks and by the diffusion coming from the dilution in the water. The general transport model, we start from, will also include possible chemical effects and radioactive decay, following the test case [4]. We study the worst possible scenario in which all units start leaking at the same time, either due to some outer factor or due to simple aging of materials used to build the containers. The ratio between the width of a single unit $l$ and the layer length $L$, can be considered as a small parameter, $\varepsilon$, in the detailed microscopic model. The study of the renormalized model behavior, as $\varepsilon$ tends to 0, by means of the homogenization method and boundary layers, gives an asymptotic model which could be used as a global repository model for numerical simulations. Parts of our work were presented in papers [1],[2],[3].

References


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Attractors for 2D-Navier-Stokes equations with delays on some unbounded domains

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We consider the following Navier-Stokes problem with delay terms:

\[
\begin{align*}
\frac{\partial u}{\partial t} - \nu \Delta u + \sum_{i=1}^{2} u_i \frac{\partial u}{\partial x_i} &= f(t) - \nabla p + g(t, u_t) \quad \text{in } (0, T) \times \Omega, \\
\text{div} u &= 0 \quad \text{in } (0, T) \times \Omega, \\
u = 0 \quad \text{on } (0, T) \times \Gamma, \\
u(0, x) &= u^0(x), \quad x \in \Omega, \\
u(t, x) &= \phi(t, x), \quad t \in (-h, 0) \quad x \in \Omega,
\end{align*}
\]

Here, the domain \( \Omega \subset \mathbb{R}^2 \) is an open set that it is not necessarily bounded but satisfies a Poincaré inequality.

Under suitable assumptions on the terms \( f \) and \( g \), existence and uniqueness of solution for this problem (and more general versions) were studied in [3]. Observe that compactness method is not valid here directly.

In this talk we will discuss on the existence of two types of attractors for some associated processes, circumventing again the lack of compact embedding (oppositely to [2]).

Our proof uses an energy method [5, 1] and is valid for the autonomous and non-autonomous cases, and in the frameworks of tempered and non-tempered universes, solving positively the invariance question for both attractors.

References


Lyapunov inequalities for differential equations with applications to nonlinear problems

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This talk is devoted to the study of $L^p$ Lyapunov-type inequalities ($1 \leq p \leq \infty$) for linear differential equations. The well-known Lyapunov inequality states that if $a \in L^\infty(0,L)$, then a necessary condition for the b.v.p.

$$u''(x) + a(x)u(x) = 0, \quad x \in (0,L), \quad u'(0) = u'(L) = 0$$

(1)

to have nontrivial solutions is: $\int_0^L a^+(x) \, dx > 4/L$, where $a^+(x) = \max\{a(x),0\}$ (see for instance [3]). Previous condition is enunciated in terms of the $L^1$-norm of the coefficients functions of the considered equation. In this talk we provide new results on $L^p$ Lyapunov-type inequalities with $1 < p \leq +\infty$ for ordinary differential equations. We show a qualitative and quantitative treatment of the problem (see [1]).

In the case of partial differential equations on a bounded and regular domains in $\mathbb{R}^N$, we present a qualitative discussion and it is proved that the relation between the quantities $p$ and $N/2$ plays a crucial role, showing a deep difference with respect to the ordinary case ([2]).

In the proof, the best constants are obtained by using variational methods, including direct methods and Lagrange multiplier theorem.

The linear study is combined with Schauder fixed point theorem to provide new conditions about the existence and uniqueness of solutions for resonant nonlinear problems. Related problems can be found in [4, 5].

References


Transmission problems related to the interaction of metallic and piezoelectric materials

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2000 Mathematics Subject Classification. 35J55, 74F05, 74F15, 74B05

We study the following mathematical problem related to engineering applications: Given is a three-dimensional composite consisting of a piezoelectric (ceramic) matrix with metallic inclusions (electrodes). We derive a linear model for the interaction of the corresponding 4-dimensional thermoelastic field in the metallic part and 5-dimensional thermoelectroelastic field in the piezo-ceramic part.

The main difficulty in the modelling is to find appropriate boundary and transmission conditions for the composed body. The mathematical analysis includes then the study of existence, uniqueness and regularity of the resulting elliptic boundary-transmission problem assuming the metallic and ceramic materials occupy smooth or polyhedral domains.

With the help of the indirect boundary integral equations method we reduce the complex transmission problem to the equivalent strongly elliptic system of pseudodifferential equations involving pseudodifferential operators on manifold with boundary. The solvability and regularity of solutions to these boundary integral equations and the original transmission problem are analyzed in Sobolev-Slobodetski (Bessel potential) $H^s_p$ and Besov $B^{s,t}_{p}$ spaces. This enables us to investigate also stress singularities which appear near zones, where the boundary conditions change and where the interface meets the exterior boundary. We show that the order of the singularity is related to the eigenvalues of the symbol matrices of the corresponding pseudodifferential operators and study their dependence on the material constants of the composite.
Analytical behavior of 2-D incompressible porous flow

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The dynamics of a fluid through a porous medium is a complex and not thoroughly understood phenomenon. In [1] we study the analytic structure of a two-dimensional mass balance equation in porous medium (PM). We obtain the existence and uniqueness of the solutions for the (PM) by the particle-trajectory method. Also, we show some blow-up criterions and some facts analogous for the two-dimensional quasi geostrophic equation thermal active scalar (QG). Detailed mathematical criteria are developed as diagnostics for self-consistent numerical calculations indicating nonformation of singularities. Finally, we obtain blow-up results in a class of solutions with infinite energy of a two-dimensional mass balance equation in porous medium.

References

On a Nonlinear Eigenvalues Problem

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We consider the problem

\[ (P^\alpha) \left\{ \begin{array}{ll}
\Delta u + \lambda (1 + u)^\alpha = 0, & \text{in } B_1; \\
u > 0, & \text{in } B_1; u = 0, & \text{on } \partial B_1
\end{array} \right. \]

where \( B_1 \) is the unit ball of \( \mathbb{R}^n \), \( n \geq 3 \), \( \lambda > 0 \) and \( \alpha > 1 \).

This problem arises in many physical models (cf. [1], [2]). It is well known (cf. [2], [3]) that there exists a constant \( \lambda^*(\alpha) \), such that \( (P^\alpha) \) admits, at least, one solution if \( 0 < \lambda < \lambda^*(\alpha) \) and no solution if \( \lambda > \lambda^*(\alpha) \). As far as the author is aware, very little is known about this constant and the corresponding solutions. Let \( \phi \) be the Lane-Emden function ([1]) in the \( n \)-dimensional space.

For every \( \rho > 0 \) and \( n \geq 3 \), one defines, \( \phi_{\rho}(r) = \frac{\phi(\rho r) - \phi(\rho)}{\phi(\rho)}, \) \( r \in [0, 1] \).

We show that, if \( 1 < \alpha \) and \( r_0 \) is the first "zero" of \( \phi \), then

\[ \lambda^*(\alpha) = \max_{r \in [0, r_0]} r^2 \phi^{\alpha-1}(r). \]

If \( 1 < \alpha < \frac{n+2}{n-2} \) and \( \lambda = \lambda^*(\alpha) \), there exists a unique \( r_{\lambda^*}(\alpha) > 0 \), such that

\[ \lambda^*(\alpha) = \left( r_{\lambda^*}(\alpha) \right)^2 \phi^{\alpha-1}(r_{\lambda^*}(\alpha)) \]

and the unique solution of \( (P^\alpha) \) is \( \psi_{r_{\lambda^*}(\alpha)} \).

When \( 0 < \lambda < \lambda^*(\alpha) \), there exists two positive constants \( r_\lambda \) and \( \rho_\lambda \), such that \( \lambda < \lambda^*(\alpha) \), there exists two positive constants \( r_\lambda \) and \( \rho_\lambda \), such that

\[
\begin{align*}
r_\lambda &= r_{\lambda^*}(\alpha) - \frac{2\lambda}{n(n-2)} - \frac{1}{\sqrt{2\lambda}} \left( \frac{1}{n(n-2)} \right)^{\frac{1}{2}} \\
\rho_\lambda &= \frac{1}{n(n-2)} \left( \frac{1}{n(n-2)} \right)^{\frac{1}{2}} \left( \frac{1}{n(n-2)} \right)^{\frac{1}{2}}
\end{align*}
\]

\( \lim_{\lambda \to 0} r_\lambda = \infty, \quad \forall \ r \in [0, 1]. \)

If \( \alpha = \frac{n+2}{n-2} \), then

\[ \lambda^*(\alpha) = \frac{n(n-2)}{4}, \quad r_{\lambda^*}(\alpha) = \sqrt{n(n-2)}. \]

\[ r_\lambda = \sqrt{1 - \frac{2\lambda}{n(n-2)}} - \frac{1}{\sqrt{2\lambda}} \left( \frac{1}{n(n-2)} \right)^{\frac{1}{2}}, \quad \rho_\lambda = \frac{1}{n(n-2)} \left( \frac{1}{n(n-2)} \right)^{\frac{1}{2}} \left( \frac{1}{n(n-2)} \right)^{\frac{1}{2}}. \]

\( \lim_{\lambda \to 0} u_\lambda = 0, \quad \lim_{\lambda \to 0} v_\lambda(r) = r^{2-n} - 1, \quad \forall \ r \in [0, 1]. \)

If \( \alpha > \frac{n+2}{n-2} \) and \( \lambda = 2 \frac{2}{(\alpha-1)^2}(\alpha(n-2)-n) \), then \( \phi(r) \sim \lambda^{\alpha-1} r^{\frac{2}{\alpha-2}}, \) as \( r \to \infty. \)
References


Identification of degenerate relaxation kernels in a heat flow with flux-type additional conditions

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An inverse problem to find degenerate time- and space-dependent relaxation kernels of internal energy and heat flux in one-dimensional heat flow is considered. The following heat equation is studied

\[ \frac{\partial}{\partial t} u(x,t) + \int_0^t n(x,t - \tau) u(x,\tau) \, d\tau = \frac{\partial}{\partial x} \left( \lambda(x) u_x(x,t) \right) \]

\[ -\frac{\partial}{\partial x} \int_0^t m(x,t - \tau) u_x(x,\tau) \, d\tau + r(x,t), \quad x \in (0,1), \ t > 0. \]

The degenerate memory kernels have the forms

\[ n(x,t) = \sum_{j=1}^{N_1} \nu_j(x) n_j(t), \quad m(x,t) = \sum_{k=1}^{N_2} \mu_k(x) m_k(t), \]

where \( \nu_j, \mu_k \) are given \( x \)-dependent functions and \( n_j, m_k \) are unknown time-dependent coefficients. Additional information to recover these kernels consists of a finite number \( N = N_1 + N_2 \) measurements of heat flux in fixed points over the time

\[ q(x_i,t) = -\lambda(x_i) u_x(x_i,t) + \int_0^t m(x_i,t - \tau) u_x(x_i,\tau) \, d\tau = h_i(t), \]

\( i = 1, \ldots, N, \ t > 0, \) where \( h_i \) are given functions.

Existence and uniqueness of a solution of the inverse problem are proved. Additional information to recover the memory kernels can consist of purely temperature observations [1, 3] or simultaneous observations of temperature and heat flux [1]. Paper [2] deal with the simplified case when the model contains only the relaxation kernel of heat flux.
References


Bifurcations in a thermoconvective problem with variable viscosity

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A Boussinesq Navier-Stokes problem with temperature-dependent viscosity in a plane parallel layer is presented. This problem has important applications in mantle convection [1, 2].

The existence of a stationary bifurcation is proved together with a condition to obtain the critical parameters at which the bifurcation takes place [3]. With appropriate changes of variables and the decomposition into normal modes we have simplified the expression to obtain a system of ordinary differential equations which is numerically manageable.

\[
\begin{align*}
\sigma \theta &= \left( D^2 - |k|^2 \right) \theta + W \\
Rk^2 \Theta &= D \left( \mathcal{P} - \frac{\sigma}{R} \right) DW - k^2 \left( \mathcal{P} - \frac{\sigma}{R} \right) W,
\end{align*}
\]

where \( \mathcal{P} f = D (\nu (z) \cdot D f) - k^2 (\nu (z) \cdot f) \), \( W \) the vertical velocity component, \( \Theta \) the temperature, \( \nu \) the variable viscosity and the boundary conditions are

\[ W = DW = \Theta = 0, \text{ on } z = \pm 1/2. \]

In reference [3] it is demonstrated that the problem (1) with the bc (2) has a unique solution \((\Theta, W) \in H_0^1 \times H_0^2\). For exponential dependence of viscosity with temperature [4, 5] the critical bifurcation curves and the most unstable modes have been numerically computed. The stability curve demonstrates coherence in the results, so for different exponential rates the critical wave number increases slightly with the exponential rate. Dependence on the exponential parameter is also presented. A higher values of the exponential rate in viscosity favors the instability.

References


Gaussian bounds for degenerate elliptic second order operators on $\mathbb{R}^n$ and the weighted Kato square root problem

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The full Kato conjecture for elliptic operators in $\mathbb{R}^n$ was finally proved in the sensational article [1]. If $L = -\text{div} (A \nabla)$ is a uniformly complex elliptic operator with bounded measurable coefficients in $\mathbb{R}^n$, then the domain of the square root operator $\sqrt{L}$ is the Sobolev space $H^1(\mathbb{R}^n)$ in any dimension with the estimate $\|\sqrt{L}f\|_2 \sim \|\nabla f\|_2$. In this short communication we will present some recent progress on the solution of the Kato square root problem for operators with degenerate ellipticity. In particular, we consider operators of the form $L_w = -w^{-1}\text{div} (A \nabla)$, where $w(x)$ is a nonnegative function in the Muckenphoupt class $A_2(\mathbb{R}^n)$, and the square matrix $A(x)$ satisfies the (degenerate) complex ellipticity condition

\[
\begin{aligned}
\lambda w(x) |\xi|^2 &\leq \text{Re} \langle A\xi, \xi \rangle = \text{Re} \sum_{i,j=1}^{n} A_{ij}(x) \xi_j \bar{\xi}_i, \\
|\langle A\xi, \eta \rangle | &\leq \Lambda w(x)|\xi \cdot \bar{\eta}|,
\end{aligned}
\]

for $\xi, \eta \in \mathbb{C}^n$ and for some $\lambda, \Lambda$ such that $0 < \lambda \leq \Lambda < \infty$. We prove that if the matrix $A$ is close to a real symmetric matrix (in the normalized $L^\infty$ norm) then the kernel of $e^{-tL_w}$ satisfies standard Gaussian bounds in $\mathbb{R}^{n+1}_+$. We will also present some positive steps towards the solution of the Kato square root problem for degenerate elliptic operators.

References

Resolution on $n$-order functional – differential equations with operator coefficients and delay in Hilbert spaces

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We introduce the $n$-order functional-differential equation with operator coefficients and delay in Hilbert spaces:

$$L^n_{p_0}U(t) = f(t), \quad D^k_t U(t) - \sum_{k=0}^{n-1} \sum_{j=0}^{m} [A_{kj} + A_{kj}(t)] S_{h_{kj} + h_{kj}(t)} D^k_t U(t) = f(t)$$

where $L^n_{p_0} : X^{n,\alpha}_{R^0_+} \rightarrow Y^{0,\alpha}_{R^0_+}, \quad D^k_t = \frac{d^k}{dt^k}, \quad R^0_+ = \{ t \geq t_0 \} \times X^{n,\alpha}_{R^0_+}$-Hilbert space, containing functions with norm:

$$|| U(t) ||^2 = \int_{t_0}^{\infty} \exp(2\alpha t) \left[ \sum_{k=0}^{n-1} || U^{(k)}(t) ||^2_y + || U^{(n)}(t) ||^2_y \right] dt, \quad t_0 \geq -\infty, \quad \alpha \in \mathbb{R}$$

$Y^{0,\alpha}_{R^0_+}$-Hilbert space, containing functions with norm:

$$|| U(t) ||^2 = \int_{t_0}^{\infty} \exp(2\alpha t) \| U(t) \|_y^2 \, dt, \quad t_0 \geq -\infty, \quad \alpha \in \mathbb{R}$$

$S_{h_{kj} + h_{kj}(t)} U(t) \triangleq U(t - h_{kj} - h_{kj}(t))$- operator translation.

Necessary and sufficient conditions for equation (1) to have unique solution are:

$$A_{kj}(t) \equiv 0, \quad h_{kj}(t) \equiv 0, \quad k \geq 0, \quad j \geq 0$$

In case the coefficients slightly variate, sufficiency is also satisfied.

References

Iterative schemes for the existence of regular solution of Korteweg’s model

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We present a Navier-Stokes type model (in velocity) with a specific stress tensor introduced for the first time by Korteweg in 1901, that takes into account that a nonuniform density (concentration or temperature) distribution induces stresses and convection in a fluid.

If the fluid is confined in a three-dimensional smooth enough domain Ω, the model can be written as:

\[
\begin{align*}
\partial_t \rho + (u \cdot \nabla) \rho &= 0 \quad \text{in } (0,T) \times \Omega, \\
\partial_t u - \nu \Delta u + u \cdot \nabla u + \nabla p &= -k \nabla \rho \Delta \rho \quad \text{in } (0,T) \times \Omega, \\
\nabla \cdot u &= 0 \quad \text{in } (0,T) \times \Omega, \\
u(t, x) \text{ is the solenoidal velocity, } p(t, x) \text{ is the fluid pressure, } \rho(t, x) \text{ is the concentration, } \nu > 0 \text{ is the fluid viscosity and } k \text{ is the capillarity coefficient.}
\end{align*}
\]

Several attempts for the study of regularity have been made. In [1], the authors study the 2-dimensional case considering an additional diffusion for the concentration \( \rho \). In [3] the existence (and uniqueness) of regular solution for the (KM) model was proved by a fixed point argument of Schauder’s type.

Here, we describe and compare two iterative methods of approximation for this model. First, we obtain some scheme estimates in regular norms (based on [2]). Secondly, we obtain the convergence from the sequence of approximate solutions towards the strong solution of (KM) by means of Cauchy’s argument in weak norms. As a consequence, the existence of regular solutions of (KM) is also proved. By the way, a priori and a posteriori error estimates are obtained.
References


Existence and uniqueness results for elliptic problems subject to mixed nonlinear boundary conditions

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In a nowadays classical paper ([2]) Brezis and Oswald introduced an elegant variational approach to prove existence and uniqueness of positive solutions to semilinear Dirichlet problems of the form

\[
\begin{cases}
\Delta u = f(x, u) & x \in \Omega \\
\frac{\partial u}{\partial \nu} = g(x, u) & x \in \Gamma, \quad u = 0 & x \in \Gamma',
\end{cases}
\]

where \( \Omega \subset \mathbb{R}^N \) is a bounded and smooth domain, \( \Gamma, \Gamma' \subset \Omega \) are smooth manifolds with a common smooth boundary, \( \partial \Omega = \Gamma \cup \Gamma' \). Assuming that \( f(x, u), g(x, u) \) are Carathéodory functions, we provide necessary and sufficient conditions for the existence of a unique solution. More interestingly, such conditions are solely expressed in terms of the main eigenvalues of certain natural associated eigenvalue problems. In this sense, this communication continues previous joint research in nonlinear boundary value problems (cf. [4, 5]).

References


Nonlinear double well Schrödinger equations in the semiclassical limit

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We consider a class of time-dependent nonlinear Schrödinger equations

\[ i\hbar \frac{\partial \psi}{\partial t} = \left[ -\hbar^2 \Delta + V \right] \psi + \epsilon W \psi, \quad \epsilon \in \mathbb{R}, \quad \psi \in L^2(\mathbb{R}^d), \]

where \( V \) is a symmetric double-well potential and \( W = W(x,|\psi|) \) is a perturbation of the type:

- global nonlinear perturbation \( W(x,|\psi|) = \langle \psi, g\psi \rangle_{L^2} g(x) \), where \( g(x) \) is a given function, in any dimension \( d \geq 1 \) [1];

- local nonlinear perturbation \( W(x,|\psi|) = |\psi|^{2\sigma} \), where \( \sigma > 0 \), in dimension \( d = 1, 2 \) ([2], [3]).

We show that, under certain conditions, the reduction of the time-dependent equation to a two-mode equation gives the dominant term of the solution with a precise estimate of the error. More precisely, in the semiclassical limit we show that the finite dimensional eigenspace associated to the lowest eigenvalues of the linear operator is almost invariant for times of the order of the beating period and the dominant term of the wavefunction is given by means of the solutions of an exactly solvable finite dimensional dynamical system.

References


Numerical study of stability properties of the Cahn-Hilliard equation

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In [1] we present stability/instability results for the so called kink solution of the Cahn-Hilliard equation. In particular the behavior of solutions of the Cahn-Hilliard equation in a neighborhood of the equilibrium is studied by exploring the Willmore functional.

The Cahn-Hilliard equation is a fourth-order reaction diffusion equation which models phase separation and subsequent coarsening of binary alloys, see [2] for a detailed description. The kink solution is a stationary solution of the equation in one dimension, connecting the two stable equilibria -1 and 1 continuously. The analogue in two dimensions are the radially-symmetric bubble solutions, compare [3].

The Willmore functional has its origin in geometry, see [4] for a discussion. A generalized version of this functional appears as the first variation of the free energy of the Cahn-Hilliard equation.

Our actual work is motivated by properties of the Willmore functional asymptotically expanded in the eigenvectors of the linearized Cahn-Hilliard operator. It can be formally concluded that the Willmore functional decreases in time for eigenvectors corresponding to a negative eigenvalue and increases in the case of a positive eigenvalue. Roughly said this means that linear instabilities which correspond to positive eigenvalues of the Cahn-Hilliard equation can be detected by considering the evolution of the Willmore functional in time. This behaviour of the Willmore functional seem to be similar for the nonlinear Cahn-Hilliard operator where of course stability needs further explanation. For nonlinear instabilities this formal conclusion is discussed in various numerical examples.

References


Decomposition method for fractional partial differential equations

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2000 Mathematics Subject Classification. 26A33, 35A, 35C, 35G, 35Q

In this paper, Adomian’s decomposition method [1] is effectively implemented for solving a class of fractional partial differential equations, where the fractional derivative based on Caputo definition. Shawagfeh [2] showed that the decomposition method is well suited to solve the nonlinear fractional differential equation:

\[ D^\alpha y(x) = f(x, y(x)), \quad x > 0, \]

where \( D^\alpha \) is the Caputo derivative of order \( \alpha > 0 \).

The aim of the present paper is to implement the Adomain decomposition method to solve initial value problems for the fractional partial differential equation,

\[ D_t^\alpha u(x, t) + L[u(x, t)] = F(x, t), \quad t > 0, \quad -\infty < x < \infty, \]

where \( D_t^\alpha \) is the Caputo fractional derivative, with respect to the variable \( t \), of order \( \alpha > 0 \), \( L \) is a partial differential operator on \( x \).

The validity of the modified technique is verified through illustrative examples including Time-fractional diffusion-wave equation, Fractional Fokker-Planck equation and Fractional Black-Scholes equation [3-5]. By this scheme, the solutions are calculated in the form of a convergent power series with easily computable components.

References

Free Boundary value problems with geometric characteristics on unknown surfaces

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The main boundary value problem studied corresponds to the axisymmetrical cavitation flow and we’ll pose it as as follows:

\[(y^{-1} \Psi_x)_x + (y^{-1} \Psi_y)_y = 0, \quad (x, y) \in D, \quad \Psi(x, y) = 0, \quad (x, y) \in \partial D,\]

\[(2y^2)^{-1} \left| \nabla \Psi(x, y) \right|^2 + \chi H + \theta K(x, y) = \lambda, \quad (x, y) \in \Sigma, \quad y > h,\]

\[\Psi(x, y) \sim \frac{y^2}{2}, \quad (x, y) \to \infty\]

Here D is the part of meridional section of axisymmetrical domain lying in the upper half plane which is also symmetrical in order of axis y. Its boundary consists of two monotone arcs, of unknown curve \(\Sigma\) lying in half space \(y > h\) connecting two points on the segment \(\{(x, y) | y = h\}\) whose endpoints coincide with those of arcs and the rest part of the segment. The letters H and K denote mean and Gauss curvature of unknown curve \(\Sigma\) respectively.

We deduce a general representation of functionals defined on \(\Sigma\) which yield Gauss curvature under its variation and formulate variational principal leading to the existence of generalized solution of the problem (1)–(3). We prove that this solution is a regular one ([1]).

The method used is further development of the method presented in the paper [2] and it can be applied for study of generalized minimal surfaces ([3]).

References

Elliptic equations with variable anisotropic nonlinearity

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We study the Dirichlet problem for the elliptic equations with variable anisotropic nonlinearity

\[-\sum_{i=1}^{n} D_i \left( a_i(x, u)|D_i u|^{p_i(x)-2} D_i u \right) + c(x,u)|u|^\sigma(x)^{-2} u = f(x), \quad (1)\]

\[-\sum_{i=1}^{n} D_i \left( a_i(x, u)|u|^\alpha_i(x) D_i u \right) + c(x,u)|u|^\sigma(x)^{-2} u = f(x) \quad \text{in } \Omega \subset \mathbb{R}^n. \quad (2)\]

It is assumed that \(p_i(x), \sigma(x) \in (1, \infty), \alpha_i(x) \in (-1, \infty)\) are given functions such that \(p_i(x), \sigma(x) \in \mathcal{C}^0(\Omega)\) with the logarithmic module of continuity, and that \(\partial \Omega\) is Lipschitz–continuous. Equations of these types emerge from the mathematical modelling of various physical phenomena, e.g., processes of image restoration, flows of electro–rheological fluids, thermistor problem, filtration through inhomogeneous media. We prove that under suitable restrictions on the coefficients and the nonlinearity exponents the Dirichlet problem for equations (1) and (2) admit a.e. bounded weak solutions which belong to the anisotropic analogs of the generalized Sobolev–Orlicz spaces and establish the classes of uniqueness of bounded solutions. The formation of the dead cores is discussed. Using a modification of the method of local energy estimates [1] we show that unlike nonlinear equations with isotropic diffusion operator, solutions of equations (1), (2) may identically vanish on a set of nonzero measure either due to a suitable diffusion–absorption balance, or because of strong anisotropy of the diffusion operator. The latter is true even in the absence of the absorption terms. The results extend to certain equations with the first–order terms and systems of equations of the structure (2). The presentation follows papers [2, 3].

References


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Error estimates for a Chernoff scheme for a nonlocal parabolic equation

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The thermistor problem has received a great interest by scientific community, and has been the subject of a variety of mathematical investigations in the past decade. We refer in particular to the works of Cimatti, Antontsev and Chipot [1], and the results by various other authors [2] and the references therein. In this work, we study a general non local parabolic equation which, under special simplifications, replaces the classical system of thermistor problem [3]. The Chernoff scheme has been studied in particular by Magenes [4], and Verdi and Visitin [5]. This numerical method consist to construct a family of time discretization schemes, where at each instant \( n \), is reduced to the resolution of a linear equation with coefficients independents of \( n \) and to a correction which consist to calculate a given function. To addition to stability results, error estimate bounds are established for a family of time discretization Chernoff scheme.

References


The communication introduces the key notions of the spectral theory of self-adjoint differential vector-operators (SADVO), which appear to be self-adjoint extensions of minimal operators generated by an Everitt-Markus-Zettl (EMZ) multi-interval differential system (see [1]). The spectral theory of SADVO was introduced in a series of works, among which we mention [2], [3] and [4]. The main topic of the communication is the method of division on subspectra (MDS), which is the principal approach in the construction of the spectral theory of SADVO. As a demonstration of the MDS, we introduce the most important constructive theorems of the spectral theory of SADVO. One of these theorems is the following generalized Mautner-Gårding-Brawder theorem (we do not give here the details on the notations used in the theorem; the theorem is given just to make an idea of the kind of problems discussed):

**Theorem** [3]. Let $T$ be a SADVO, generated by an EMZ system $\{I_i, \tau_i\}_{i \in \Omega}$. Let $U$ be an ordered representation of the space $L^2 = \oplus_{i \in \Omega} L^2(I_i)$ relative to $T$ with the measure $\theta$ and the multiplicity sets $s_k$, $k = 1, \ldots, m$. Then there exist kernels $\Theta_k(\vee x_i, \lambda)$, measurable relative to $d(\vee x_i) \times \theta$, such that $\Theta_k(\vee x_i, \lambda) = 0$ for $\lambda \in \mathbb{R} \setminus s_k$ and $(\oplus_{i \in \Omega} \tau_i - \lambda)\Theta_k(\vee x_i, \lambda) = 0$ for each fixed $\lambda$. Moreover for any bounded Borel set $\Delta$,

$$\int_{\Delta} |\Theta_k(\vee x_i, \lambda)|^2 d\theta(\lambda) \in \oplus_{i \in \Omega} L^\infty(I^n_i) \forall n = 1, 2, \ldots$$

$$(Uw)^k(\lambda) = \lim_{n \to \infty} \int_{I^n} w(\vee x_i) \overline{\Theta_k(\vee x_i, \lambda)} d(\vee x_i), \quad w \in L^2,$$

where the limit exists in $L^2(s_k, \theta)$. The kernels $\{\Theta_k(\vee x_i, \lambda)\}_{k=1}^n$, $n \leq m$, are linearly independent as vector-functions of the first variable almost everywhere relative to the measure $\theta$ on $s_n$.

**References**


On the dynamics of a Klein-Gordon-Schrödinger type system

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2000 Mathematics Subject Classification. 35B30, 35B40, 35B45, 35B65, 35D10, 35L70, 35Q53, 35Q55

We present certain contemporary trends in the theory of Klein-Gordon-Schrödinger type Systems. Then we give some recent results on the existence, uniqueness and asymptotic behaviour of solutions for the following evolution system of Klein-Gordon-Schrödinger type

\begin{align*}
  i\psi_t + \kappa \Delta \psi + i\alpha \psi &= \phi \psi, \\
  \phi_{tt} - \Delta \phi + \phi + \lambda \phi_t &= -\text{Re}\psi_x, \\
  \psi(v,0) &= \psi_0(v), \phi(v,0) = \phi_0(v), \phi_t(v,0) = \phi_1(v), \\
  \psi(v,t) &= \phi(v,t) = 0, \quad v \in \partial \Omega, \quad t > 0,
\end{align*}

where \( x \in \Omega, \ t > 0, \ \kappa > 0, \ \alpha > 0, \ \lambda > 0 \) and \( \Omega \) is a bounded subset of \( \mathbb{R}^N \), with \( N \leq 3 \). This certain system describes the nonlinear interaction between high frequency electron waves and low frequency ion plasma waves in a homogeneous magnetic field. We prove the existence of a global attractor in the strong topology of the space \( (H^1_0(\Omega) \cap H^2(\Omega))^2 \times H^1_0(\Omega) \) which attracts all bounded sets of \( (H^1_0(\Omega) \cap H^2(\Omega))^2 \times H^1_0(\Omega) \) in the norm topology and establish certain energy decay estimates under some parametric restrictions. Finally, we are going to present some upper estimates for the Hausdorff and Fractal dimensions of the global attractor.

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References


On a discrete magnetic Laplacian on differential forms

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2000 Mathematics Subject Classification. 35Q60, 39A12, 39A70

Let \((M,g)\) be a Riemannian manifold, \(\text{dim}M = n\). Denote \(\Lambda^p(M)\) the set of all differentiable complex-valued \(p\)-forms on \(M\). We define a magnetic potential as a real-valued 1-form \(A \in \Lambda^1(M)\). We will need a deformed differential

\[
d_A : L^2(M) \rightarrow L^2\Lambda^1(M), \quad \varphi \rightarrow d\varphi + i\varphi A,
\]

where \(i\) is the imaginary unit and \(d\) is the differential. Let \(\delta_A\) be the formal adjoint operator of the operator \(d_A\), i.e. \(\delta_A : L^2\Lambda^1(M) \rightarrow L^2(M)\). Let us identify the magnetic potential \(A\) with the multiplication operator \(A : L^2(M) \rightarrow L^2\Lambda^1(M), \varphi \rightarrow \varphi A\). Then the formally adjoint operator \(\delta_A\) can be written as follows

\[
\delta_A\omega = (\delta - iA^\ast)\omega,
\]

where \(\delta, A^\ast\) are the formal adjoint operators of \(d\) and \(A\) respectively. Using (1),(2), we can write the magnetic Laplacian \(-\Delta_A \equiv \delta_A d_A : L^2(M) \rightarrow L^2(M)\) as follows

\[
-\Delta_A \varphi = -\Delta \varphi - iA^\ast d\varphi + i\delta(A\varphi) + A^\ast A\varphi,
\]

where \(-\Delta \equiv \delta d\). Operator (3) is essentially self-adjoint (see [2]).

The main purpose of this talk is to construct an intrinsically defined discrete model of the magnetic Laplacian. Speaking about this discrete model we do not mean just the corresponding difference operator on a lattice but we mean a discrete analog of the Riemannian structure on some combinatorial object. We consider discrete forms as certain cochains with complex-valued components. We define a discrete analog of the exterior multiplication operation, a combinatorial Hodge star operator and an inner product of discrete forms. We also construct discrete analogs of the operators (1)-(3) acting on some finite-dimensional Hilbert spaces.

Our approach bases on the formalism proposed by Dezin [1]. In the spirit of [1] we study self-adjointness of the discrete magnetic Laplacian and we prove that the Dirichlet problem for the discrete Poisson type equation has a unique solution.
One of the formal results is the construction of a nonstandard approximation of the generalized solution of the Poisson type equation (with the magnetic Laplacian (3)) under the minimal requirements of smoothness of the right hand side, see also [3].

References


Two-scale homogenization for evolutionary variational inequalities via the energetic formulation

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This paper is devoted to the homogenization for a class of rate-independent systems described by the energetic formulation. The associated nonlinear partial differential system has periodically oscillating coefficients, but has the form of a standard evolutionary variational inequality. Thus, the model applies to standard linearized elastoplasticity with hardening.

Using the recently developed methods of two-scale convergence, periodic unfolding and the new introduced one, periodic folding, we show that the homogenized problem can be represented as a two-scale limit which is again an energetic formulation, but now involving the macroscopic variable in the physical domain as well as the microscopic variable in the periodicity cell.

References


Distribution of subharmonic and ultraharmonic waves in the presence of nonlinear mechanism of the energy dissipation

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The statics and dynamics of mechanical systems with nonlinear mechanism of the energy dissipation which is a contact dry friction is a new scientific direction in nonlinear mechanics of a deformable firm body.

In case of dynamic deformation in nonlinearity of the dissipation’s mechanism represents a priori unknown sign-nonlinearity of required function’s speed. In case of movement an unknown sign function takes plus or a minus unit values and at stops it may take any values between plus and a minus unit and this value should be fixed in task solution process. Such kind of tasks come to consideration of the hyperbolic type equation’s nonlinear system and are connected with consideration of distribution and attenuation of nonlinear waves.

The first complexity is in the definition of the expression of the friction’s sign-function, which essentially depends on both boundary and initial conditions and its dry friction low. The area of dependence of the decision in such problems is defined by a method of kappa-function of Nikitin-Tyurekhodjaev.

Specificity of these problems is that nonlinearity does not allow to generalize the results of one task to a class of tasks. Analytical results in the distribution of nonlinear waves in the system with contact dry friction under the influence of cyclic loads are obtained by the authors.

Based on the results of obtained decisions the following conclusion has been made: there is a class of cyclic loads with the frequency to arbitrary integer times which differs from the frequency of its own fluctuation of system under the action of which the system will make the established subharmonic or ultraharmonic fluctuations with two frequencies. Besides, one fluctuation coincides with the frequency of its own fluctuation of system, the other - with the frequency of external loading.
References


Singularities of Bernoulli free boundaries

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A Bernoulli free-boundary problem [4] is one of finding domains in the plane on which a harmonic function simultaneously satisfies linear homogeneous Dirichlet and inhomogeneous Neumann boundary conditions. One of the earliest, and still one of the most important, of such problems comes from Bernoulli’s theorem and the constant-pressure condition in the study of Stokes waves in hydrodynamics.

In this talk based on the paper [5] we show that, for a large class of Bernoulli problems, a free boundary which is symmetric with respect to a vertical line through an isolated singular point must necessarily have a corner at that point, and we give a formula for the contained angle. The assumptions used admit the possibility of other singular points, even uncountably many, on the free boundary. This result is a substantial extension of that in the theory of hydrodynamic waves [1, 3] on the existence of a corner of 120° at the crest of symmetric Stokes waves of extreme form. We also show that, even in the presence of singularities, any geometrically simple Bernoulli free boundary is necessarily symmetric, extending the result for the water-wave problem in [2].

References

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Existence of a Palais-Smale sequence with nonzero limit

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In this talk, we are going to introducing a new method to construct a Palais-Smale Sequence. Moreover, we are going to find a condition for a domain such that in which a Palais-Smale Sequence admits a nonzero limit. The nonzero limit turns out to be a nonzero solution of the associate semilinear elliptic equation.

References

Two-fold completeness of root vectors and well-posedness of hyperbolic problems

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2000 Mathematics Subject Classification. 47A, 34L, 35L

We consider a spectral problem for a system of second order (in the spectral parameter) abstract pencils in a Hilbert space and prove the completeness of a system of eigenvectors and associated vectors; obtain, in some special cases, the expansion of vectors with respect to eigenvectors; consider a relevant application of these abstract results to boundary-value problems for second order ordinary differential equations with a quadratic spectral parameter in both the equation and boundary conditions.

We also study an initial boundary-value problem for hyperbolic differential-operator equations and give a relevant application to hyperbolic PDEs with the same (second) order of differentiation with respect to the time in both the equation and boundary conditions. For the proof we use some special transformations and some results from [1].

For example, the following problem is treated for completeness questions:

\[ L(\lambda)u := \lambda^2 u(x) + \lambda A_1 u(\cdot) - (b(x)u'(x))' + B_1 u(\cdot) = 0, \quad x \in [0, 1], \]

\[ L_1(\lambda)u := \alpha \lambda^2 u(0) + u'(0) = 0, \quad L_2(\lambda)u := \beta \lambda^2 u(1) + u'(1) = 0, \]

where \( \alpha < 0, \beta > 0 \) (note that \( A_1 \) can be, e.g., a multiplication operator and \( B_1 \) can be the first order differential operator with variable continuous coefficients) and the following problem is treated for hyperbolic PDEs:

\[ L(D_t)u := D_t^2 u(t, x) + a_1(t, x)D_x^2 u(t, x) + a_2(t, x)D_t u(t, x) - D_x (b(x)D_x u(t, x)) + B_1(t)D_t u(t, \cdot) = h(t, x), \quad (t, x) \in [0, T] \times [0, 1], \]

\[ L_1(D_t)u := \alpha D_t^2 [u(t, 0)] + D_x u(t, 0) = h_1(t), \quad t \in [0, T], \]

\[ L_2(D_t)u := \beta D_t^2 [u(t, 1)] + D_x u(t, 1) = h_2(t), \quad t \in [0, T], \]

\[ u(0, x) = u_0(x), \quad D_t u(0, x) = u_1(x), \quad x \in [0, 1], \]

where \( \alpha < 0, \beta > 0, D_t := \frac{\partial}{\partial t}, D_x := \frac{\partial}{\partial x}. \) By \( D_t^2 [u(t, c)] \) we mean \( \frac{d^2 u(t, c)}{dt^2} \), where \( c = 0, 1. \)
References

$W^{1,p}$-solvability of partial differential relations

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2000 Mathematics Subject Classification.

We study the solvability of the partial differential relation $\nabla u(x) \in K$ for $u \in W^{1,p}(\Omega; \mathbb{R}^m)$, where $\Omega \subset \mathbb{R}^n$ is a bounded domain, $\nabla u(x)$ is the Jacobi matrix of $u$ and $K$ is a given set of $m \times n$ matrices. Let $U$ be another set of $m \times n$ matrices. We say that $U$ is $W^{1,p}$-reducible to $K$ if there exists a constant $c(p,U,K) > 0$ such that, for some bounded open set $\Omega \subset \mathbb{R}^n$ with $|\partial \Omega| = 0$ and for every $\xi \in U$, $\epsilon > 0$, there exists a function $v \in \xi x + W^{1,p}_0(\Omega; \mathbb{R}^m)$ satisfying two conditions: (i) $v = \sum_{i \in \mathbb{N}} v_i \chi_{\Omega_i}$, where $\{\Omega_i\}$ is a family of disjoint open subsets of $\Omega$ with $|\Omega \setminus \bigcup_{i \in \mathbb{N}} \Omega_i| = 0$ and $v_i$ satisfies $\int_{\Omega_i} |Dv_i(x)|^p \, dx \leq c(p,U,K) \, |\Omega_i|$ and, either $v_i|_{\Omega_i} = \xi x + b_i$ with $\xi_i \in U$ or $Dv_i(x) \in K$ a.e. $x \in \Omega_i$ for each $i$; (ii) $\int_{\Omega} \text{dist}(Dv(x); K) \, dx < \epsilon |\Omega|$. We say that $U$ is uniformly locally $W^{1,p}$-reducible to set $K$ if for each $\xi \in U$ there exists a bounded set $U_\xi \subset U$ containing $\xi$ that is $W^{1,p}$-reducible to $K$ with constant $c(p,U_\xi,K) \leq C(1 + |\xi|^p)$, where $C = C(p,U,K) \geq 1$ is a uniform constant independent of $\xi$.

The main result of this paper is the following existence theorem proved in [5].

**Main Theorem:** Let $1 < p < \infty$ and let $U$ be uniformly locally $W^{1,p}$-reducible to a closed set $K$. Then, for any piecewise affine function $\varphi \in W^{1,p}(\Omega; \mathbb{R}^m)$ with $D\varphi(x) \in U \cup K$ a.e. $x \in \Omega$ and for any $\epsilon > 0$, there exists a solution $u \in W^{1,p}(\Omega; \mathbb{R}^m)$ to the relation $\nabla u(x) \in K$ a.e. in $\Omega$ which satisfies

$$
\|u - \varphi\|_{L^p(\Omega)} < \epsilon; \quad \int_{\Omega} |Du|^p \leq C \left( |\Omega| + \int_{\Omega} |D\varphi|^p \right),
$$

where $C = C(p,U,K) \geq 1$ is a uniform constant.

The method of proof relies on a different Baire’s category argument from [1] concerning the residual continuity of a Baire-one function used by [2]. Some sufficient conditions for $W^{1,p}$-reduction are also given along with certain generalization of some known results [3] and a specific application to the boundary value problem for special weakly quasiregular mappings [4].
References


Convergence to equilibrium for nonlinear evolution equations

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In a series of papers [1], [2], [3] jointly with H. Wu and M. Grasselli, we have extended Lojasiewicz-Simon approach to some nonlinear evolution equations with dissipative boundary conditions and obtained the results on convergence of the solution to an equilibrium as time goes to infinity. These nonlinear evolution equations include:
1. The following Cahn-Hilliard equation

\[
\begin{cases}
\frac{\partial u}{\partial t} = \Delta \mu, \\
\mu = -\Delta u - u + u^3,
\end{cases}
\quad (x, t) \in \Omega \times \mathbb{R}^+ \tag{1}
\]
with dissipative boundary condition

\[
\begin{cases}
\sigma_s \Delta ||u - \partial_\nu u + h_s - g_s u = \frac{1}{\Gamma_s} u_t, \\
\partial_\nu \mu = 0,
\end{cases}
\quad t > 0, \ x \in \Gamma \tag{2}
\]
and the initial condition.
2. The following damped semilinear wave equation with critical exponent

\[
u_{tt} + u_t - \Delta u + f(x, u) = 0,
\quad (x, t) \in \Omega \times \mathbb{R}^+ \tag{3}
\]
subject to the dissipative boundary condition

\[
\partial_\nu u + u + u_t = 0,
\quad t > 0, \ x \in \Gamma \tag{4}
\]
and the initial conditions.
3. The following hyperbolic=parabolic phase-field system

\[
\begin{cases}
(\theta + \chi)_t - \Delta \theta = 0, \\
\mu \chi_{tt} + \chi_t - \Delta \chi + \phi(\chi) - \theta = 0,
\end{cases}
\quad (x, t) \in \Omega \times \mathbb{R}^+, \tag{5}
\]
with dissipative boundary conditions

\[
\begin{cases}
\partial_\nu \theta = 0, \\
\partial_\nu \chi + \chi + \epsilon \chi_t = 0,
\end{cases}
\quad (x, t) \in \Gamma \times \mathbb{R}^+, \tag{6}
\]
and the initial conditions.
References


Self-similar solutions for the 1-D Schrödinger map on the hyperbolic plane

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We study the self-similar solutions developing corners in finite time for the following evolution equation:

$$X_t = X_s \times X_{ss}.$$  \hfill (1)

Here, $X(s,t)$ represents a curve parameterized with $s$ and $\times$ is the pseudo-cross product according to the Minkowski-metric:

$$a \times b = (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, -(a_1 b_2 - a_2 b_1)).$$

We prove that the solutions corresponding to the initial data

$$X(s,0) = A^+ s \chi_{[0,+\infty)}(s) + A^- s \chi_{(-\infty,0]}(s),$$  \hfill (2)

develop a finite-time corner-like singularity. In (2), $A^+$ and $A^-$ are vectors in the unit sphere in $\mathbb{R}^{2,1}$. This problem admits a reformulation in terms of the 1-D Schrödinger equation through Hasimoto's transformation.

An analogous situation has been considered before in the Euclidean case by Gutiérrez, Rivas and Vega [3] in connection with the evolution of vortex filaments. The main difference of our work with respect to the Euclidean case studied by these authors is the proof of the boundedness of the generalized trihedron $T$, $e_1$ and $e_2$ in $\mathbb{R}^{2,1}$.

References


On the local well-posedness for some systems of coupled KdV equations

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Using the theory developed by Kenig, Ponce, and Vega, we prove that the Hirota-Satsuma system is locally well-posed in Sobolev spaces $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ for $3/4 < s \leq 1$. Using the results obtained by Christ, Colliander, and Tao, we show ill-posedness for the Hirota-Satsuma system in $H^s(\mathbb{R}) \times H^{s'}(\mathbb{R})$ for $-1 \leq s < -3/4$ and $s' \in \mathbb{R}$. Moreover, we present some comments to scale changes performed in some previous papers concerning the Gear-Grimshaw system. We introduce some Bourgain-type spaces $X^a_{s,b}$ for $a \neq 0$, $s,b \in \mathbb{R}$, and we prove some properties for these spaces. Finally, we obtain local well-posedness for the Gear-Grimshaw system in $H^s(\mathbb{R}) \times H^s(\mathbb{R})$ for $s > -3/4$, by establishing new mixed-bilinear estimates involving the two Bourgain-type spaces $X^{-\alpha}_{s,b}$ and $X^{\alpha}_{s,b}$ adapted to $\partial_t + \alpha_- \partial_x^2$ and $\partial_t + \alpha_+ \partial_x^3$ respectively, where $|\alpha_+| = |\alpha_-| \neq 0$.

References