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Short Communications Abstracts

Section 19

Mathematics Education and Popularization of Mathematics

A Popperian characterization of mathematics: a reference for the teacher

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In this contributed paper we present the main ontological, epistemological and methodological aspects of mathematics deriving from K. R. Popper's philosophical conception. We take them as a departure for the elaboration of a theoretical frame from where reflecting on the teaching-learning of mathematics.

We take special care in underlining an interpretation of Popper's ontological position, according to which mathematics is created, but at the same time it is discovered. Attention is paid to the way how mathematical knowledge is developed, starting with the interrelation between the knowing subject and the mathematical object in the context of mathematical or scientific community. In addition, we revise his definition of mathematical proof, with the idea that this is something that cannot be reduced to mere axiomatic nexuses.

The synthesis of all these aspects leads us to the conception of the practice of mathematical teaching-learning starting from the (re)contextualization or (re)creation of mathematical knowledge, through the resolution of problems with the assistance of technological supports. So, the development of the curriculum by means of the unidirectional transmission of theorems already available is passed to a second term. In this way, mathematical teaching and training is centred in the practice of the community of experts.

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Mathematics meets the public: prizes, exhibitions and conferences in Catalonia and the rest of Spain

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Public access to mathematics in a non formal education setting takes many different routes, including newspapers and magazines, books, radio, television, the Internet, and films or theater. Each of these media has different needs, strengths and effectiveness for the popularisation and learning of mathematics [1, 2, 3]. This report is the first outcome of a project intending to review and analyse the state of the art in Catalonia and Spain during the last ten years, and specifically analyses the initiatives of museums, libraries and cultural institutions or associations in the form of exhibitions, prizes, homages and courses. Results show an insufficient number of links between research institutions and science communication centers, which leads to the selection and display of content that is not as up to date as desirable. On the other hand, we find a plethora of under-funded and under-staffed initiatives with great potential, usually initiated by non-lucrative groups of teachers with meager publicity resources and scarce impact.

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How to make the best of multiple choice tests

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A good way for checking the students' understanding of the concepts they are taught and their ability to use them is an open test. However, there are many cases when one has to use multiple choice tests. By a multiple choice problem we mean a problem with five or more answers where only one of the answers is correct; this answer has to be marked by the student.

Lecturers who teach a large number of students are often forced to use multiple choice exams. Some lecturers even prefer multiple choice tests because they provide objective grading.

On the other hand, many teachers are confident that with multiple choice tests we can check only the calculating ability of a student.

Based on our experience in teaching mathematics to engineering students, we construct examples from calculus that show how one can get around many of the disadvantages of multiple choice tests, and check the students' knowledge of the theory including proofs of theorems

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Maths, magic and media

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n this talk we present a few mathematical magic tricks involving elementary mathematics, originally made as a performance in the framework of "Science on Stage" program and later to be played in a science museum. The presented tricks are based on ideas exposed in the mathemagical literature, mainly in the papers/books by Álvarez et al. [1], Gardner [2] and Mulcahy [3]. In the second part of the talk we comment different reactions developed by people attending the performance as well as the interest of media in this activity. Communication media (newspapers, radios, TV) have shown a great interest in this unusual way to transmit mathematical culture to the society.

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Is mathematics recognized as relevant in economics and management courses?

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Worldwide it is increasingly recognized that mathematics is a foundation of many science fields including in the social and management sciences, where it is also seen as a necessary tool to formalize logical and economic thinking. Thus, potential effect on course content becomes a fundamental issue concerning the role of mathematics in the training of economists and managers in terms of the expression and communication. At University of Eduardo Mondlane of Mozambique, the last curriculum reform changed the position of mathematics in most Faculties, and in Economics apart of reducing the time spent on teaching mathematics it was also decided to apply a more application-oriented teaching approach, where each new concept has been illustrated by examples and problems related to a variety of practical-situations in related fields. This adjustment leaded to a redefinition of the contents and the teaching approaches for mathematics and consequently contributed to improve its relationship with other subjects in the curriculum. Though, to evaluate the impact of these changes at Economics Faculty, where students were involved in a "learning mathematics by doing", a questionnaire was administered to access the perception on the importance and usefulness of mathematics in their career and the effect of the teaching approach on the students' performance. Although the small differences found, the general results showed that for the majority of those students, Mathematics still be recognized as highly important and relevant and that it can not be acceptable to study the current economics without a strong background in mathematics.

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What students think and why it matters - a survey of student teachers' views of mathematics

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As the international view of teaching has shifted from didactic to constructivist with its image of learner as participatory, so research on teacher education has moved from a focus on the transfer of a body of knowledge to a more dynamic view of the classroom, with teachers being facilitators of learners' knowledge construction. In this view of teaching, teacher beliefs and attitudes play an important role in shaping classroom practice (see [1]) and there is a substantial body of evidence examining this supposed link between teachers' attitudes to and beliefs about mathematics and teaching, and classroom practice (see [2], [3], [4] and [5]).

This paper reports the initial findings of a survey of attitudes to mathematics among students in Northern Ireland training to teach in primary schools (pupils aged 4 to 11 years) at the beginning of their initial teacher training programmes. It compares the attitudes of those taking a postgraduate course with those undertaking an undergraduate degree. It uses multidimensional definitions of attitude and belief and examines the responses of the students to a number of statements concerning mathematics. In asking the students to reflect on these responses at the beginning of their course it is hoped to develop an awareness in the students of the complexities of the interaction between classroom practice and attitudes and beliefs.

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Toward improving secondary teachers' understanding of mathematical problem solving via rich task design

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Mathematics teachers are under intense pressure in the United States to develop their students' competency in mathematical problem solving. If we are to have secondary mathematics teachers that are successful in teaching mathematical problem solving to their students, then they must first become successful problem solvers in their own right ([5], [4]). The author developed a Mathematical Task Enrichment Guide (MTEG) for practicing secondary mathematics teachers to use in the design of rich mathematical tasks from ordinary textbook problems. The use of the MTEG challenged teachers' understanding of mathematical problem solving requiring them to re-examine their beliefs about mathematics teaching ([3]). Also, MTEG requires the unlearning of a linear view of problem solving toward a cyclic view ([2],[5]). Findings suggest that teachers' understanding of mathematical problem solving improved after they used the MTEG and that their beliefs about the nature of mathematical problem solving shifted toward a view held by effective problem solvers ([1]).

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Understanding children talking mathematics: analysis of communication in the virtual problem solving environment CAMI

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In our talk, we will discuss about children talking mathematically and how we, the teachers, could better understand their thinking through communication.

The last five years were marked by a profound shift of New Brunswick's francophone school system toward new teaching and learning paradigms along with other Canadian provinces. Under strong influence of NCTM standards from one side, and new didactical approaches based on competencies from another side, the new mathematics curriculum puts emphasis on communication, use of technology and critical thinking in a genuine problem-solving environment. But how well are teachers equipped to deal with diversity of strategies and means of communication created by children?

CAMI (a French abbreviation for Chantier d'Apprentissages Mathématiques Interactifs) is a mathematical communication between K-12 schoolchildren and pre-service teachers in which schoolchildren are invited to solve challenging mathematical problems that are being posted on the Internet site (using a model of the Problem of the Week of the MathForum, at mathforum.org) and to submit their solutions electronically. University students then evaluate each solution and send a personal comment to each child. A general comment on the problem with samples of interesting solutions and names of children that succeeded appears on the WEB site www.umoncton.ca/cami two weeks after the problem is posted. Since 2000, the first year of its creation, the project is constantly expanding involving more and more participants, mostly from New Brunswick.

Using several examples from this mathematical communication, we will analyze several patterns of children's mathematical talk and pre-service teacher's response.

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Experience of didactic innovation in algebra. A didactic unit on the affine function

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This paper presents an action research on the effects produced on mathematics teachers and students by the use of innovative Algebra curricular material. The didactic innovation consisted in the design, validation and application of a Didactic Unit on the affine function.

The theoretical basis of this research can be found in Ausubel's significative learning within the constructivist epistemology and in the guidelines established by Rico, Callejo and other authors concerning the methodology for Didactic units in high school mathematics.

The research's methodology went through two stages. The first stage involved design and application of the Didactic Unit to seven groups of high school students from both public and private schools. The second stage comprised the analysis of the different experiences, the redesign of the document and the application to other two groups, after considering the contributions of each teacher.

The importance of the results obtained lies in the effort to analyze this experience. The results influenced: curricular development through Didactic Units; evaluation in mathematics to benefit learning; constructivist significative learning; competition in mathematics among students; professional improvement of teachers; quality of the contents in school Algebra; role of participants in the educational process and use of diverse methodologies in the classroom.

The impact of this research goes beyond the participants' experiences as results can be applied to other educational areas and contexts. For example, a methodology for the design of innovative material, an action research model for the professional improvement of teachers and the optimization of the didactic material.

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Lesson study

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Lesson study is a process of collaboratively planning, teaching, observing, revising, and reporting results from one class lesson. Last fall, a group of mathematics faculty at UW-Stout participated in a workshop on lesson study led by Bill Cerbin of UW-La Crosse. We decided to do a project on a related rates lesson in Calculus I. This talk will include results from our lesson study, as well as tips for doing your own lesson study project.

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Teachers beliefs about proof in the Spanish education system

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In the past ten years, extensive research on the role of proof in the learning and teaching of mathematics has been carried out in opposition to the previous emphasis placed in heuristics(Hanna, 2000). The methodological approach proposed in Principles and Standards at NCTM (2000) echoes it.

On the other hand a great number of papers deal with the complex and direct connection that exists between the values and beliefs of the teacher and his actual teaching practice in the classroom. Therefore, in this project, the purpose is to analyse the essence of the views of the Spanish secondary education teacher on the nature of mathematics, and his teaching. Particularly, his views on the role of proof in the practice of teaching.

For this purpose, first some available secondary sources have been synthesised. Secondly, a qualitative research has been carried out with discussion groups, following the guidelines of the social studies methods.

The result show that, in the past few years, there has been an intentional disregard of the deductive aspects of mathematics in order to favour the problem-solving approach. With the findings of this study, I would like to foster wider research about the consequences of the different conceptions of teachers and the need of a new methodological approach to the teaching of proof in the Spanish education system.

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Discrete multifractals

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A p-adic multifractal formalism developed by the author [1] can provide an arena for multifractal analysis that is accessible to advanced undergraduates. The theory provides a way to investigate functions $f : N^d \to N$ with regard to their "order" of divisibility by powers of a prime p. For example, it can be shown that the number of integer pairs (i, j) with $0 \leq i, j < p^n$ for which the binomial coefficient $\binom{i+j}{i}$ is divisible by p^{zn} but not p^{zn+1}, where zn is an integer and $0 \leq a < z < b \leq 1$, is asymptotically $p^{nD(a,b)}$, where $D(a, b) := \sup\{D(z) : a < z < b\}$ and $D(z)$, the "multifractal spectrum" of dimensions, is given by an explicit, elementary function. Furthermore, the p-adic theory relates this to a partition function through a Fenchel-Legendre transform. The theory provides a framework for students to investigate other combinatorial functions, either by rigorous analysis or computer exploration.

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Math on-line education: state of the art, experiences and challenges

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In the first part of this article, we make a general review about the state of the art of the on-line mathematical education, or mathematical e-learning, in the university environment [2, 5]: centers from different countries that offer this type of courses, methodological aspects and tendencies, employed technologies (web platforms, mathematical software and Internet resources), materials, main problematics and results, integration with classical lectures, acceptance among faculty staff and students, future perspectives, etc. The previous review is followed by a second part in which we describe and analyze the methodological aspects, and some personal experiences, linked to our teaching activity, developed during the last ten years at the Open University of Catalonia -a completely on-line university, in subjects from the mathematical and statistical knowledge area. Special attention is paid to math curricula in computer science degrees [1, 3], where a lot of work can be done in order to adapt math courses to both the continuously changing necessities of computer science students and, also, to the challenges that the European Higher Education Space brings into this knowledge area [4].

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Pedagogical ideary of mathematics' teachers: a historical-cultural perspective

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The purpose of this research is to answer the question: how do the pedagogical ideary and the teaching practice of pre-service teachers of Mathematics (re)constitute themselves in a process of education mediated by action, reflection and investigation about the pedagogical practice? The objectives conducting this study are: to identify the constituent elements of the pedagogical ideary that is being produced by pre-service Math teachers about Mathematics, and its learning and teaching processes, and about the teaching practice in general; to investigate how this ideary and the teaching practice of pre-service Math teachers are problematized and (re)constitute themselves in a process of action, reflection and investigation about the pedagogical practice in Mathematics; and to investigate the relationship established between the process of ideary (re)constitution in pre-service teachers and their teaching practice. Based on historical-cultural theoretical references on the subjects and their constitution, I approach the movement of the ideary (re)constitution and of the teaching practice of three pre-service teachers of Mathematics based on their daily lives. This approach was accomplished in the courses Teaching Practice and Supervised Teaching Apprenticeship I and II, taught at the program Mathematics Teachers' Education, at the UNICAMP (Brazil). The road followed in this study was supported by the research method narrative inquiry. The records and data analyzed were produced by those three pre-service teachers from autobiographies, case analyses, written texts, reflexive diaries, conceptual maps, and interviews. I identified, among other things, some of the constituent elements of their pedagogical ideary and this term was (re)signified.

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Take cognizance of comprehensive mathematics: ethnomathematics teacher education

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Ethnomathematics [1] appears with organized groups that joined in on cultural, ideological, religious or professional basis since the primary stages of group organization. For some nations, ethnomathemtical awareness was part of the values imparted along the generations. Hence, Ethnomathematics, which is worthwhile for representing the understanding of mathematics as a multi-varied one, can facilitate the multi-cultural approach in math education, while mathematical contents could be "presented to learners from different cultures in various ways" [2] and become relevant to the cultural variety in math classroom [3]. In this presentation, I am proposing a program in ethnomathematics learning and it's interweaving in school mathematics' teaching.

Math teachers, Jews and Arabs, secular and religious, who learn in Kaye College, form a rich mosaic of backgrounds for the growth of mathematical-socio-cultural dialogue on mathematics education. The study that was based on Grounded Theory approach which enables to build a theory as mutual reaction between the data for their processing, examined the aspects of educational process undergone by mathematics teachers and the potential benefits that can stem from training teachers in ethnomathematics. The findings suggest two prevalent tendencies in ethnomathemtical training of teachers: "systemic enhancement", by enrichment of mathematics teacher training program and creation of learning environment with orientation toward multi-cultural education in mathematics, and "teacher's qualities", by improvement of mathematics understanding, acquirement of math teaching skills in cultural perspective and creation of linkage between the teacher to his people and roots.

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Various degrees of the number's distinction

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We present new, elementary concepts suggested by observations of how children learn mathematics. The concepts are geometric in nature. Euclidean geometry is based on the axiom that through every point outside a given line there is exactly one line parallel to that given line. But, as every student of mathematics knows, we obtain great benefit from imagining systems in which this axiom is not true. Exploiting our studies of how children learn mathematics, we suggest an augmentation of the concept of number. Young children see numbers simultaneously as both ordinal and cardinal. We discovered by working with children that there is yet another dimension to their conception of numbers that we call the degree of distinction. We relate this observation to other observations such as the fact that children think of lines as atomistic objects rather than as sets of points. Through these investigations, we arrive at a new meta-conception of number in which, among other things, number becomes a bridging between the continuum and the discreetness. In so doing we believe that we have discovered a way to view number which permits a great diversity of conceptual models for number, even when those conceptions seem to fundamentally clash. This observation enables us further research about the difficulties young children have, in understanding the concept of Infinity.

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Influences of the teachers' beliefs and strategies in the teaching-learning process of Math: A Constructivist Solution Proposal

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The role of the teacher is that of a mediator or facilitator; their main task is to help the students build up new knowledge or meaning from what they already know, and from how they are and how they learn. Therefore, the teachers' knowledge and beliefs regarding the teaching and learning of Math will influence the way in which their students learn.

Although in the representations, knowledge and beliefs are integrated, they may be psychologically differentiated. Knowledge has, essentially, a social and cultural character, and satisfy a criteria of what is true and acceptable. Beliefs, that have an individual and subjective character (Thompson, 1992) are understandings and premises about the world, perceived as true, and imply cognoscible and affective personal codes (Richardson, 1996). Beliefs are product of the environment in which the teaching of Math takes place, and influence the way in which it is learned and applied.

Considering all this, the objectives of the research were to analyze the beliefs of the teachers towards the teaching and learning of Math, together with the analysis of their teaching strategies. We used an adapted M^{c}Combs and Wilser (1997) scale, and a set of open questions.

The results are discussed in terms of the beliefs of the teachers, their teaching-learning strategies, and the implications in their teaching practices. The proposal includes a program of continuous education, without incrementing the workload of the teacher. The research took place in the Republic of Panama.

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Education issue on the dawning of randomness

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During the last thirty five years I have had the opportunity to realize, many times, how worldwide are the damages to society due to the lack of knowledge and smart use of probabilistic mathematics and holistic view. Values of: motivation, revenues and knowledge are beeing lost. Increased costs are being generated as well. The previous telecommunications, entertainment and information networks are converging to the borning multimidia New Generation Network. To reach their goals with the highest levels of clients satisfaction and productivity, all network management "end to end" processes need active cooperation between two main approaches: stochastic and holistic from one side and deterministic and reducionist on the other. Some of the damages mentioned above, concerning networks management, are the result of weak cooperation between those views; but going deeper I have found that the lack of probabilistic and holistic education is the main root cause. I would like to ilustrate this issue with one of the last cases I have participated in.

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Martin Gardner, Alice and the law of gravity

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The main goal of this presentation is to describe an alternative proposal, which integrates Literature, Philosophy and Mathematics, intended to facilitate and motivate the classroom introduction of some scientific concepts, from Mathematics and Physics, such as the law of gravity, among others.

Alice in Wonderland is a universally well-known story, which, as a motivating tool, lets us exhibit contextualized scientific fundamental concepts, where literature, history and fantasy are present. Those concepts have been traditionally introduced into the classroom in a formal but cold way. Here, we begin with Alice's fall into a profound well through the earth's center, in order to explain gravity's behavior according to the chosen model used to describe the internal structure of our planet. We mention some descriptions of the earth's inner part, proposed in [3] such as: the mythological Hades, Dante's Hell, Julio Verne's fantasies and the novel Pellucidar by Edgar Rice Burroughs.

Since the work of Lewis Carroll is made up from absurd or non-sense, it permits us the incursion in a quite variety of complex problems. The Carroll's non-sense purpose is to question our beliefs, those that, because of repetition, became unquestionable truths. Carroll's work leads us to knowledge, to feel curiosity and fascination for things around us, and specially gives us the possibility to make our brain work in a creative way.

The authors of this essay (which can be found at [2]), have exchanged discipline views converging to a pedagogical proposal, summarized in the presentation of a problem concerning gravity, in three directions: Literature, Mathematics and Philosophy. Through this vision, we hope that the students, at college level, improve their capacity to analyze and develop their cultural background.

Our purpose is to show how velocity and time once reaching the center of the earth vary according to the reference model we use. We remark, by the way, that, the values found differ from those that Martin Gardner exhibits in his book The Annotated Alice [1].

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A new generation of cubic surface models: retrieving the Clebsch

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In the second half of the nineteenth century, the interest of mathematicians in algebraic geometry grew enormously. One of the reasons was the fascinating discovery of Salmon and Cayley in 1849: *Any smooth cubic surface contains precisely 27 lines.* Due to the great development, mathematical models were built in order to illustrate geometrical properties of some surfaces. One of these surfaces was the Clebsch diagonal surface, first defined by Clebsch in 1871. The Clebsch is a smooth cubic surface with the property that all its 27 lines are real. This surface has been built several times in the history of time. First in plaster (by Clebsch in 1872 and by the German firms Brill and Schilling. More recently, the sculptors Claudia Carola Weber and Ulrich Forster built the surface in clay (see [3]).

When a surface is built in plaster or in clay, the interior has to be filled in, in order to make the result solid. In this way, some of the mathematical properties may not be appreciated. As a consequence, mathematicians who work in this area nowadays use different kinds of software, see for example [2].

In this work, we present a polyester sculpture of the Clebsch surface and of two other cubic surfaces with singularities, with the purpose of fusing a precise mathematical visualization and the artistic representation. We also recall some classical mathematical properties of cubic surfaces [1], and discuss a more modern approach [4]. In this way, we aim to help the popularization of this particular area of mathematics.

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The Best Method to Teach Calculus to the Non-mathematical Branches in the University

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Mathematics is very important in many branches in the university including Economics, Management, etc. , in which it is used as a tool to express economical theories. Calculus is the basic course in economics and students have to learn 11 units in Calculus I, Calculus II, and Math III (Calculus and Differential and Difference Equation). In Iran there are two ways of teaching calculus to the students in non-mathematical branches like Economics. In one system, an Economic student enrolls in the course with the other students who may be studying in Mathematic or other non-mathematical branches. In this type of classes the lecturer assumes certain equal background and interest for all students and teaches pure Mathematics. The lecturer usually dose not discuss the application of what she/he is teaching in the other area of study. The exercises are pure mathematics and the students field of study may not be consider or even not known. In the second method, the students of Economics enroll in a course that is specially designed for them. All the students are studying in the same field and the lecturer is aware of their field of study, therefore, after teaching each section she/he will discuss examples that show the application of the course materials in Economics. In this research we show that the students re more interested in the second method and more successful than the students who learned calculus with the first method. A questioner was distributed among the students at the end of semester and the result was extracted using Inference Statistics methods. The result of this research can be used for the other university branches like Management, Sociology, etc.

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Conceptual articulation of calculus: constraction of the tangent line without the use of derivative

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A fundamental issue in calculus learning is the introduction of the tangent line concept. The traditional approach has been to make use of limits and derivative. It is a tool of the construction of the tangent line equation that can - and often is - used without a proper understanding of the underlying ideas, thus propitiating a conceptual disarticulation. Here we propose an alternative method, as is applicable to the fundamental notions of calculus: growth, decrease, critical points, concavity, and slope of the tangent line. The purpose is to aid students in developing a better understanding of these basic notions and, importantly, of their interrelations.

First, a correlation between the construction of the tangent line and the identification of the local maximum and minimum points is established [1], [2]. Subsequently, to keep the precalculus level and to avoid the introduction of derivative, the slope m of the tangent line is obtained by analysis of inequalities that are obtained from the definitions of critical points. This method has a general character and is applicable to different types of functions. Some simple examples are offered.

This material will be useful for students at the K-12 level in addition to methods of analysis of functions students on undergraduate level.

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Teaching applied statistics at UPC: integrating lectures, statistical software and e-learning

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The article presents a pedagogical model, which has been designed for teaching applied statistics at the School of Building Construction of Barcelona in the Technical University of Catalonia (UPC).

The pedagogical model is based on the use of software technology and defines activity sequences in an integrated project in order to motivate learning, guide reasoning and, finally, improve knowledge and skills acquisitions [1].

The model is based on the integration of three aspects: (a) lectures, where statistical concepts and applications are introduced to students, (b) laboratory practices, in which students make use of statistical software to solve activities related with the concepts introduced during lectures, and (c) development of a statistical project, linked to the technical architecture environment [3, 4, 5], that small groups of students carry out in several phases with the help of statistical software. For the project development, students make use of the UPC web platform, so that students can do online collaborative work and can receive assistance from the faculty staff. These three aspects of the model interrelate among them, giving place to an educational model that integrates traditional lectures with information technologies (statistical software and web platforms) [2], and promotes both students' active participation and collaborative learning.

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From the Riemann integral to the Fundamental theorem of calculus: an approach with applet Descartes

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This work presents a graphic approach to the concept of Integrals to make the student gradually transiting from the sums of Riemann to the study of integral as a superior end function, and its characteristics. A tracer of the Integral Function builded with the applet Descartes, is used to get visually into the relationship between a function and its integral function; this is, the Fundamental Theorem of Calculus. This is going to take advantage of the software capabilities to allow the students interacting in a graphical and numerical way with computers in order to prove their results, to predict characteristics and to guess about general situations. The use of this kind of computerized resources, usable also in internet, where the student manipulates graphs dynamically, is very helpful before to get into an abstract approach. This proposal is framed in the context of the new educative model that is implemented today in the Universidad de Sonora.

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Some curiosities about cabri: analitic geometry as a tool for teacher training

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At the Universidad Autónoma de Querétaro courses on analytical geometry, at the baccalaureate, are taught by people whose prime formation is not as mathematicians. Motivated by the need for updated courses as well as for skilled teachers, efforts are done to develop new didactic strategies for analytical geometry courses.

In this note we are concerned with teacher training for analytical geometry courses. Traditionally, analytical geometry is based on the concept of a plane whose coordinate axes form a 90^o angle: Why not to consider a different angle introducing in this way several coordinate frames?

In particular, using new technologies and the software CABRI Geometre II, we deal with questions such as:

- How does the formula used to compute distance is modified if the coordinate axes are not orthogonal?
- How does the concept of slope changes?
- What happens with the straight line equation?
- What can be said about the different geometric loci?

We try to make teachers to became familiar with the mathematical rationale by questioning several fundamental concepts of analytical geometry in order to gain more insight on them.

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