Rings whose non-zero finitely generated modules are retractable

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We give several equivalent formulations of a finite retractable ring which is defined to be a ring $R$, all of whose non-zero finitely generated (right) modules $M$ are retractable, in the sense that $\text{Hom}_R(M, N) \neq 0$ for any non-zero submodule $N$ of $M$. One such formulation involving matrix rings over $R$ states that if $I$ is any right ideal in $S = \text{Mat}_{n \times n}(R)$ and $x \in S \setminus I$, then there exists $s \in S$ such that $xs \notin I$ and $xsI \subseteq I$. Initially, retractable modules appeared in [3] and then in connection with the study of endomorphism rings, being Baer, CS, quasi Baer, etc. in [1],[2],[4]. More recently, P. F. Smith [5] characterized retractable modules over right FBN rings. In this paper, we use our characterizations of finite retractable rings to show that the class $\mathcal{C}$ of these rings contains any ring that is Morita equivalent to a commutative ring, and that if $R$ is a right order in $T$, then $R \in \mathcal{C}$ implies that $T \in \mathcal{C}$. Finally, a finitely annihilated module $M$ over a finite retractable ring is shown to be a weak generator in $\sigma[M_R]$.