Norm equalities for operators

V. Kadets, M. Martín and J. Merí*

Departamento de Análisis Matemático, Universidad de Granada, Facultad de Ciencias
18071 Granada, Spain [jmeri@ugr.es]

2000 Mathematics Subject Classification. 46B20

Our purpose is to study equalities involving the norm of operators on Banach
spaces, and to discuss the possibility of defining isometric properties for Banach
spaces by requiring that all operators of a suitable class satisfy such a norm equality.

The interest in this topic goes back to 1963, when I. Daugavet [1] showed that
each compact operator \( T \) on \( C[0,1] \) satisfies the norm equality

\[
\| \text{Id} + T \| = 1 + \|T\|. \tag{DE}
\]

The above equation is nowadays referred to as Daugavet equation. A Banach space
is said to have the Daugavet property if (DE) holds for every rank-one operator
[2]. This property has been subject of intense investigation and it has deep con-
sequences on the geometry of a Banach space. We study whether it is possible to
define other properties by requiring that all rank-one operators on a Banach space
satisfy an equation of the form

\[
\|g(T)\| = f(\|T\|) \tag{1}
\]
or

\[
\|\text{Id} + g(T)\| = f(\|g(T)\|), \tag{2}
\]

where \( g \) is analytic and \( f \) is continuous. For the first case, we prove that the Dau-
gavet property is the only non-trivial possibility. For the second case, a property
different from the Daugavet property appears and we show that only these two
possibilities may happen in the complex setting.

[1] Daugavet, I. K., On a property of completely continuous operators in the space \( C \),