Excesses and deficits of frames in shift-invariant subspaces

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For an invertible $n \times n$ matrix $B$ and $\Phi$ a finite or countable subset of $L^2(R^n)$, consider the collection

$$X = \{ \phi(\cdot - Bk) : \phi \in \Phi, k \in Z^n \},$$

generating the closed subspace $\mathcal{M}$ of $L^2(R^n)$. Let $T_{\mathcal{F}(X)}(\xi)$ denote the frame operator associated with the frame $\{ \mathcal{F}\phi(\xi) \}_{\phi \in \Phi}$ defined for a.e. $\xi \in [0,1)^n$, where $\mathcal{F}$ is the isometric isomorphism between $L^2(R^n)$ and $L^2(T^n, \ell^2(Z^n))$. Using a very nice property of the range function, the Gramian and dual Gramian operators ($G$ and $\tilde{G}$ resp.) and $\mathcal{F}$, we will show that if $\mathcal{M}$ is a Shift-Invariant subspace generated by $X$, one need at most $m$ functions, where $m = \| \dim(Ker(\tilde{G}(\cdot))) \|_\infty$, to generate the orthogonal complement of $\mathcal{M}$ in $L^2(R^n)$. Furthermore, if $k \geq m$ or $k = \infty$, one can always find $k$ functions such that the associated Shift-Invariant system form a Parseval tight frame for $\mathcal{M}^\perp$. Finally we will show that the existence of a collection of $m$ sequences in the orthogonal complement of the range of analysis operator associated with the frame $X$ that satisfies any of four interesting conditions is equivalent to $\dim(Ker(G(\xi)))$, the dimension of the kernel of Gramian operator, being less than or equal to $m$ for almost all $\xi \in [0,1)^n$.