Representations of simple finite Lie conformal superalgebras of type $W$ and $S$

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Lie conformal superalgebras encode the singular part of the operator product expansion of chiral fields in two-dimensional quantum field theory [1]. The list of finite simple Lie conformal superalgebras is much richer, mainly due to existence of several series of super extensions of the Virasoro conformal algebra. The complete classification of finite simple Lie conformal superalgebras was obtained in [2]. The list consists of current Lie conformal superalgebras $\text{Cur} \, g$, where $g$ is a simple finite-dimensional Lie superalgebra, four series of “Virasoro like” Lie conformal superalgebras $W_n$ ($n \geq 0$), $S_{n,b}$ and $\tilde{S}_n$ ($n \geq 2, b \in \mathbb{C}$), $K_n$ ($n \geq 0$), and the exceptional Lie conformal superalgebra $CK_6$.

The main result of the present communication is the construction of all finite irreducible modules over the Lie conformal superalgebras $W_n, S_{n,b}$ and $\tilde{S}_n$. The proof relies on the observation that representation theory of Lie conformal superalgebras is controlled by the representation theory of the (extended) annihilation superalgebra. This reduces to the study of certain representations of the Lie superalgebra $W(1,n)_+$ of all vector fields on a superline (an affine superspace of dimension $(1|n)$) and the Lie superalgebra $S(1,n)_+$ of such vector fields with zero divergence. As in [3], [4], we follow the approach developed for representations of infinite-dimensional simple linearly compact Lie algebras by A. Rudakov in [5]. The problem reduces to the description of the so called degenerate modules, and for the later we have to study singular vectors.