Some results about the totally positive completion problem

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A partial matrix over \( \mathbb{R} \) is an \( n \times n \) array \( A = (a_{ij}) \) in which some entries are specified real numbers while the remaining (unspecified) entries are free to be chosen from \( \mathbb{R} \). A partial matrix is said to be combinatorially symmetric when the \((i, j)\) entry is specified if and only if the \((j, i)\) entry is, and is non-combinatorially symmetric in the other case. A completion of a partial matrix is a choice of values for the unspecified entries resulting a conventional matrix. A matrix completion problem asks which partial matrices have a completion with some desired property. The specified positions in an \( n \times n \) non-combinatorially symmetric partial matrix \( A = (a_{ij}) \) can be represented by a directed graph \( G_A = (V, E) \), where the set of vertices \( V \) is \( \{1, 2, \ldots, n\} \) and \((i, j), i \neq j, \) is an arc of \( E \) if the \((i, j)\) entry is specified.

An \( n \times n \) real matrix \( A = (a_{ij}) \) is said to be totally positive (nonnegative) if every minor is positive (nonnegative). These matrices are having an increasing importance in approximation theory, combinatorics, statistic, economics, computer aided geometric design and wavelets, etc.

Total nonnegativity is inherited by submatrices. Therefore, it is a necessary condition that every fully specified submatrix be totally nonnegative. A partial matrix that satisfied this necessary condition is said to be partial totally nonnegative. Here we are interested in the totally nonnegative completion problem, that is, when a partial totally nonnegative matrix has a totally nonnegative completion. This problem was studied for combinatorially symmetric partial matrices by Johnson, Kroschel and Lundquist in [1] and by Jordán and Torregrosa in [2], and by el-Ghamry, Jordán and Torregrosa for the non-combinatorially symmetric case in [3].

In this work we completed the study done in [3]. We get necessary and sufficient conditions in order to obtain a totally nonnegative completion of a partial totally nonnegative matrix, whose associated graph is a path, a cycle, a totally specified path, a double-path, etc.

