The ring $C(X)$ modulo its socle and principal ideals

**F. Azarpanah*, O. A. S. Karamzadeh and S. Rahmati**

Department of Mathematics, Chamran University, Ahvaz, Iran [azarpanah@ipm.ir]

2000 **Mathematics Subject Classification.** 54C40

The essentiality of prime ideals and $z$-ideals of $C(X)$ modulo the socle and principal ideals of $C(X)$ are investigated. We show that the Goldie dimension of $C(X)$ modulo its socle is greater than or equal to that of $C(X)$ and the equality may not always hold. Topological spaces $X$ for which the Jacobson radical of $C(X)$ modulo its socle is zero are characterized. Using this characterization, it turns out that for a compact space $X$, the Jacobson radical of $C(X)$ modulo its socle is zero if and only if the set of isolated points of $C(X)$ is finite.
