Stability for generators of cosine functions

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We prove the stability of rational methods associated to certain one-step time discretization of abstract well-posed second-order in time problems

\[
\begin{aligned}
\begin{cases}
    u''(t) &= Au(t), \quad t \in \mathbb{R}, \\
    u(0) &= u_0 \\
    u'(0) &= v_0.
\end{cases}
\end{aligned}
\]

where the differential operator \( A \) generates a cosine function on a Hilbert space \( H \).

Our first step is to transform (1) into a first-order in time system. For this, we define the operator \( A : D(A) = D(A) \times X \subset X \times X \rightarrow X \) given by

\[
A = \begin{bmatrix}
0 & I \\
A & 0
\end{bmatrix},
\]

and we consider the abstract initial value system:

\[
\begin{aligned}
\begin{cases}
    u'(t) &= Au(t), \quad t \in \mathbb{R}, \\
    u(0) &= u_0
\end{cases}
\end{aligned}
\]

where \( u(t) = [u(t), v(t)]^T, \ t \in \mathbb{R} \) and \( u(0) = [u_0, v_0]^T \).

The rational methods that we study are suitable for these problems and they can be defined, for example, by using Runge-Kutta-Nyström methods which form a wider class than the Runge-Kutta ones. The stability in the energy norm when applied to (2) has only been studied in the literature by us in [1], where we study the case of Runge-Kutta-Nyström methods for the selfadjoint case and finite dimensional problems.

For our results, we use functional calculus for operators with numerical range contained in a parabola [2].

We remark that we will restrict the study to the Hilbertian case. Although the Banach space case is well studied in the literature, the most relevant examples are only well-posed for the Hilbertian case.